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Adapted Riccati technique and non-oscillation of linear and half-linear equations

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Abstract

The aim of this paper is to mention a generalization of the adapted Riccati equation and, using this method, to prove a non-oscillatory result concerning half-linear differential equations with coefficients having mean values. Note that this result is new even for linear equations.

Keywords: Riccati technique, p-Laplacian, half-linear equation, non-oscillation criterion, Riccati equation, oscillation theory, linear differential equation
2010 MSC: 34C10, 34C15

1. Introduction

The classic Riccati technique is one of the most widely used methods in the oscillation theory of differential equations. In this paper, we intend to present a process which is based on the classic Riccati technique, but developed to be applicable to more general and complicated problems. Especially, we are interested in the so-called half-linear equations also known as the (partial) differential equations with the one dimensional p-Laplacian. For a comprehensive theory and literature overview concerning half-linear equations, we refer to the well-known book [3].

Our research is motivated mainly by paper [2], where a new modification of the Prüfer angle is discussed. Note that this modified Prüfer angle covers several of its variations (see, e.g., [1, 4, 10, 11]). The Riccati and Prüfer methods can be combined to obtain results about complicated equations, but the Prüfer method became more developed than the Riccati one (due to [2]). Since historically (according to our experience and knowledge) the Riccati technique was used more often and many important results were obtained via this method, our goal is to illustrate the fact that the Riccati method is applicable to equations (as, e.g., in [7]) which are in the center of interest of researchers today.

After the simple description of the method, we demonstrate its usage by proving a non-oscillation criterion. A method of the proof can be considered as a separate technique of applications of the presented adapted generalized Riccati technique. It covers several of the mentioned papers although the presented main result itself is not a consequence of any known result. For the basic motivation, we refer to papers [1, 6, 12, 13]. In papers [5, 14, 15, 16], similar problems are treated and the research is going in another direction there.

This paper is organized as follows. In Section 2, we clarify the notion of the adapted generalized Riccati equation itself. Then, in Section 3, we obtain a new non-oscillation criterion for a certain type of half-linear equations.

2. Adapted generalized Riccati equation

In this section, we mention the generalized Riccati technique. Let $p > 1$ be given and let q be the number conjugated with p, i.e., $p + q = pq$. We consider the second-order half-linear differential equation

$$\left[t^{\alpha} r^{1-p}(t)\Phi(x')\right]' + t^{\alpha-p} s(t)\Phi(x) = 0, \qquad \Phi(x) = |x|^{p-1} \operatorname{sgn} x, \tag{2.1}$$

where $t \in \mathbb{R}$ is sufficiently large, $\alpha \in [0, p-1)$, and r, s are continuous functions having mean values and

$$0 < R_1 := \inf\{r(t); t \in \mathbb{R}\} \leq R_2 := \sup\{r(t); t \in \mathbb{R}\} < \infty. \tag{2.2}$$

Let x be a non-zero solution of Eq. (2.1). Using $\zeta(t) = -\Phi\left(\frac{tx'(t)}{r(t)x(t)}\right)$, we obtain

$$\zeta'(t) = \frac{1}{t}\left[(p-\alpha-1)\zeta(t) + s(t) + (p-1)r(t)|\zeta(t)|^q\right]. \tag{2.3}$$

Based on similar cases in the literature, Eq. (2.3) is called the adapted generalized Riccati equation. For details of the derivation above, we refer to [8] (and also to [3, Eq. (9.2.3) on p. 439]).

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