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# MAXIMUM PRINCIPLE AND ITS APPLICATION FOR THE NONLINEAR TIME-FRACTIONAL DIFFUSION EQUATIONS WITH CAUCHY-DIRICHLET CONDITIONS

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Abstract. In this paper, a maximum principle for the one-dimensional sub-diffusion equation with Atangana-Baleanu fractional derivative is formulated and proved. The proof of the maximum principle is based on an extremum principle for the Atangana-Baleanu fractional derivative that is given in the paper, too. The maximum principle is then applied to show that the initial-boundary-value problem for the linear and nonlinear time-fractional diffusion equations possesses at most one classical solution and this solution continuously depends on the initial and boundary conditions.

## 1. Introduction and statement of problem

In this paper, we consider the nonlinear time-fractional diffusion equation

$$\frac{\partial u}{\partial t}(x,t) = \frac{\partial^2}{\partial x^2} D_{*t}^{1-\alpha} u(x,t) + F(x,t,u) \ \ in \ \ (0,a)\times(0,T] = \Omega, \tag{1.1}$$

with the following nonhomogeneous Cauchy-Dirichlet conditions

$$\begin{cases} u(x,0) = \varphi(x),\ x\in[0,a], \\ u(0,t) = \lambda(t),\ \ u(a,t) = \mu(t),\ 0\le t\le T, \end{cases} \tag{1.2}$$

where the functions $F(x,t,u), \varphi(x), \lambda(t), \mu(t)$ are continuous and $\lambda(t), \mu(t)$ are nondecreasing functions, $D_{*t}^{\alpha}$ is the Atangana-Baleanu fractional derivative (see Section 2).

The aim of this paper is to research the maximum principle for the nonlinear fractional diffusion equation (1.1). We first introduce some recent related works. Maximum principles were given in [4, 5, 6, 7, 14] for the types of fractional diffusion equations different from (1.1). For the maximum principles given in [6] to hold, existence of a regular solution (with existence of a solutions $u_t$ on the closed time interval $[0,T]$) is assumed. In [7], the assumption of a solution with existence of a continuous $u_t$ in $(0,T]$ such that $u_t \in L^1([0,T])$ is made. In [8] Ahmad, Alsaedi and Kirane studied three types of fractional diffusion equations. For each type, they obtained an upper bound of the Chebyshev norm in terms of the integral of the solution.

In [9] Chan and Lui was considered a maximum principle for the equation (1.1) where Riemann-Liouville derivative is considered rather that instead Atangana-Baleanu derivative [11, 12, 13].

If $\alpha \to 0$ then equation (1.1) by Property 2.3 coincides with the classical heat equation. The equation of the form (1.1) with fractional derivatives with respect to the time variable is called the sub-diffusion equation [1]. This equation describes the slow diffusion.

The nonlinear problem (1.1) and (1.2) having a solution implies $u_t(x,t)$ exists. Thus for any $0<\alpha<1$. $D_t^{1-\alpha}u(x,t)$ exists for $t>0$. Hence, a solution $u(x,t)$ of the problem (1.1) and (1.2) in the region $[0,a]\times[0,T]$ is a (classical) solution in $C([0,a]\times[0,T]) \cap C^{2,1}((0,a)\times(0,T])$.

## 2. Some definitions and properties of fractional operators

In this section, we recall some basic definitions and properties of the fractional derivative operators.

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