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An operator splitting strategy for fluid-structure interaction problems with thin elastic structures in an incompressible Newtonian flow★

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Abstract

We present a computational framework based on the use of the Newton and level set methods to model fluid-structure interaction problems involving elastic membranes freely suspended in an incompressible Newtonian flow. The Mooney-Rivlin constitutive model is used to model the structure. We consider an extension to a more general case of the method described in [1, Appl Math Lett 69 (2017) 138–145] to model the elasticity of the membrane. We develop a predictor-corrector finite element method where an operator splitting scheme separates different physical phenomena. The method features an affordable computational burden with respect to the fully implicit methods. An exact Newton method is described to solve the problem, and the quadratic convergence is numerically achieved. Sample numerical examples are reported and illustrate the accuracy and robustness of the method.

Keywords: Newton method, Operator splitting, Navier-Stokes flow, Embedded interface, Finite Element Method.

1. Introduction

The individual and collective behaviors of red blood cells (RBCs) strongly influence of the rheological properties of blood. The cytoskeleton (inner filamentous network of proteins) is responsible for the mechanical properties of RBCs and helps the cell to withstand hydrodynamic stresses without breaking. Understanding the RBC's mechanics provides a basis for predicting the hemorheology [2]. The modeling of RBCs entails the resolution of a difficult fluid-structure interaction (FSI) problem involving thin and highly deformable structures freely suspended in an incompressible fluid. This framework concerns the development of stable and efficient methodology to solve such a problem. At the numerical level, the large deformations of membranes in flow may induce several instability issues and immediately rule out a wide-set of numerical methods as candidates to describe such a dynamics, e.g the Arbitrary Lagrangian Eulerian methods [3]. These instabilities are exacerbated for problems involving extremely slender and light structures in incompressible flow. Embedded interface-type approaches are more appropriate to solve such a problem [3, 4, 5, 6].

In this work we present a stable predictor-corrector numerical method based on the use of the Newton method and operator splitting algorithm to model elastic membranes in incompressible Newtonian flow.

2. Mathematical formulation

Let $T > 0$ and $d = 2, 3$ be the simulation period and space dimension, respectively. For any time $t \in (0, T)$, let $\Gamma(t)$ be an elastic membrane. The symmetric part of any tensor $\boldsymbol{T}$ is denoted by $\mathbf{sym}(\boldsymbol{T}) = \boldsymbol{T} + \boldsymbol{T}^T$. The tensorial product and the two times contracted product between tensors are denoted by the symbols "$\otimes$" and ":", respectively.

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