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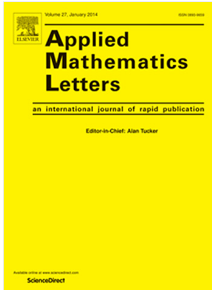


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# Second critical exponent for a nonlinear nonlocal diffusion equation *

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**Abstract.** This paper studies the Cauchy problem for a nonlocal diffusion equation $u_t = J * u - u + u^p$. We determine the second critical exponent, namely the optimal decay order for initial data at space infinity to distinguish global and blow-up solutions in the "super Fujita" range. Similar to the critical Fujita exponent obtained very recently by Alfaro[1], we find that the second critical exponent also relies heavily on the behavior of the Fourier transform of the kernel function $J$.

**Keywords**: Nonlocal diffusion; Global existence; Blow-up; Second critical exponent

## 1 Introduction

Consider positive solutions of the Cauchy problem for a nonlinear nonlocal diffusion equation

$$\begin{cases} u_t = J * u - u + u^p, & (x,t) \in \mathbb{R}^N \times (0,T), \\ u(x,0) = u_0(x) = \lambda\varphi(x), & x \in \mathbb{R}^N, \end{cases} \tag{1.1}$$

where $*$ stands for the usual convolution, and $\varphi$ is a nonnegative and bounded function in $\mathbb{R}^N$. In this paper, we assume that $J$ is a nonnegative, bounded, radial and satisfies $\int_{\mathbb{R}^N} J\,dx = 1$. Moreover, we assume that there exist $0 < \beta \le 2$, $m > N$, $A > 0$ and $C > 0$ such that

$$\hat{J}(\xi) = 1 - A|\xi|^\beta + o(|\xi|^\beta) \quad as \quad |\xi| \to 0, \tag{1.2}$$

and

$$|\hat{J}(\xi)| \le \frac{C}{|\xi|^m}, \quad as \quad |\xi| \to \infty. \tag{1.3}$$

The assumptions of $J$ are the same as that in [6] where the linear nonlocal diffusion equation $u_t = J * u - u$ is studied. In particular, if $J$ is compactly supported or $J$ satisfies (1.2) with $[\beta] > \frac{N}{2}$, then (1.3) holds for any $m > N$ and some $C = C(m) > 0$, see Theorem 1.14 in [2].

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