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Second critical exponent for a nonlinear nonlocal diffusion equation *

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Abstract. This paper studies the Cauchy problem for a nonlocal diffusion equation $u_t = J * u - u + u^p$. We determine the second critical exponent, namely the optimal decay order for initial data at space infinity to distinguish global and blow-up solutions in the “super Fujita” range. Similar to the critical Fujita exponent obtained very recently by Alfaro[1], we find that the second critical exponent also relies heavily on the behavior of the Fourier transform of the kernel function J .

Keywords: Nonlocal diffusion; Global existence; Blow-up; Second critical exponent

1 Introduction

Consider positive solutions of the Cauchy problem for a nonlinear nonlocal diffusion equation

$$\begin{cases} u_t = J * u - u + u^p, & (x, t) \in \mathbb{R}^N \times (0, T), \\ u(x, 0) = u_0(x) = \lambda \varphi(x), & x \in \mathbb{R}^N, \end{cases} \quad (1.1)$$

where $*$ stands for the usual convolution, and φ is a nonnegative and bounded function in \mathbb{R}^N . In this paper, we assume that J is a nonnegative, bounded, radial and satisfies $\int_{\mathbb{R}^N} J dx = 1$. Moreover, we assume that there exist $0 < \beta \leq 2$, $m > N$, $A > 0$ and $C > 0$ such that

$$\hat{J}(\xi) = 1 - A|\xi|^\beta + o(|\xi|^\beta) \text{ as } |\xi| \rightarrow 0, \quad (1.2)$$

and

$$|\hat{J}(\xi)| \leq \frac{C}{|\xi|^m}, \text{ as } |\xi| \rightarrow \infty. \quad (1.3)$$

The assumptions of J are the same as that in [6] where the linear nonlocal diffusion equation $u_t = J * u - u$ is studied. In particular, if J is compactly supported or J satisfies (1.2) with $[\beta] > \frac{N}{2}$, then (1.3) holds for any $m > N$ and some $C = C(m) > 0$, see Theorem 1.14 in [2].

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