### **Accepted Manuscript**

Second critical exponent for a nonlinear nonlocal diffusion equation

Jinge Yang

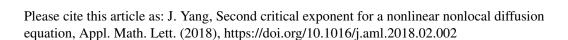
PII: S0893-9659(18)30031-4

DOI: https://doi.org/10.1016/j.aml.2018.02.002

Reference: AML 5430

To appear in: Applied Mathematics Letters

Received date: 12 December 2017 Accepted date: 5 February 2018



This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# Second critical exponent for a nonlinear nonlocal diffusion equation \*

Jinge Yang<sup>†</sup> School of Sciences, Nanchang Institute of Technology Nanchang 330099, P. R. China

December 12, 2017

**Abstract.** This paper studies the Cauchy problem for a nonlocal diffusion equation  $u_t = J * u - u + u^p$ . We determine the second critical exponent, namely the optimal decay order for initial data at space infinity to distinguish global and blow-up solutions in the "super Fujita" range. Similar to the critical Fujita exponent obtained very recently by Alfaro[1], we find that the second critical exponent also relies heavily on the behavior of the Fourier transform of the kernel function J.

Keywords: Nonlocal diffusion; Global existence; Blow-up; Second critical exponent

#### 1 Introduction

Consider positive solutions of the Cauchy problem for a nonlinear nonlocal diffusion equation

$$\begin{cases} u_t = J * u - u + u^p, & (x,t) \in \mathbb{R}^N \times (0,T), \\ u(x,0) = u_0(x) = \lambda \varphi(x), & x \in \mathbb{R}^N, \end{cases}$$
 (1.1)

where \* stands for the usual convolution, and  $\varphi$  is a nonnegative and bounded function in  $\mathbb{R}^N$ . In this paper, we assume that J is a nonnegative, bounded, radial and satisfies  $\int_{\mathbb{R}^N} J \, dx = 1$ . Moreover, we assume that there exist  $0 < \beta \le 2$ , m > N, A > 0 and C > 0 such that

$$\hat{J}(\xi) = 1 - A|\xi|^{\beta} + o(|\xi|^{\beta}) \quad as \quad |\xi| \to 0,$$
 (1.2)

and

$$|\hat{J}(\xi)| \leq \frac{C}{|\xi|^m}, \quad as \quad |\xi| \to \infty. \tag{1.3}$$

The assumptions of J are the same as that in [6] where the linear nonlocal diffusion equation  $u_t = J * u - u$  is studied. In particular, if J is compactly supported or J satisfies (1.2) with  $|\beta| > \frac{N}{2}$ , then (1.3) holds for any m > N and some C = C(m) > 0, see Theorem 1.14 in [2].

<sup>\*</sup>Supported by the STRP of Jiangxi Province (GJJ161112), NNSF of China (11701260).

<sup>&</sup>lt;sup>†</sup>Corresponding author. E-mail: jgyang2007@yeah.net

#### Download English Version:

## https://daneshyari.com/en/article/8053797

Download Persian Version:

https://daneshyari.com/article/8053797

<u>Daneshyari.com</u>