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# Direct sampling method for anomaly imaging from scattering parameter

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ABSTRACT

In this paper, we develop a fast imaging technique for small anomalies located in homogeneous media from scattering parameter data measured at dipole antennas. Based on the representation of scattering parameters when an anomaly exists, we design a direct sampling method (DSM) for imaging an anomaly and establishing a relationship between the indicator function of DSM and an infinite series of Bessel functions of integer order. Simulation results using synthetic data at $f = 1$ GHz of angular frequency are illustrated to support the identified structure of the indicator function.



## 1. Introduction

In this study, we consider an inverse scattering problem that determines the locations of small anomalies in a homogeneous background using scattering parameter (or simply, $S$-parameter) measurements. This study has been motivated by microwave tomography for small-target imaging, such as in the case of tumors during the early stages of breast cancer. Because of the intrinsic ill-posedness and nonlinearity of inverse scattering problems, this problem is very hard to solve; however, it is still an interesting research topic because of its relevance in human life. Many researchers have focused on various imaging techniques that are mostly based on Newton-type iteration-based techniques [1, Table II]. However, the success of Newton-type based techniques is highly dependent on the initial guess, which must be close to the unknown targets. Furthermore, Newton-type based techniques have various limitations such as large computational costs, local minimizer problem, difficulty in imaging multiple anomalies, and selecting appropriate regularization. Because of this reason, developing a fast imaging technique for obtaining a good initial guess is highly required. Recently, various non-iterative techniques have been investigated, e.g., MUltiple SIgnal Classification (MUSIC) algorithm, linear sampling method, topological derivative strategy, and Kirchhoff/subspace migrations. A brief description of such techniques can be found in [2–6].

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Direct sampling method (DSM) is another non-iterative technique for imaging unknown targets. Unlike the non-iterative techniques mentioned above, DSM requires either one or a small number of fields with incident directions [7–10]. Furthermore, this is a considerably effective and stable algorithm. Due to this reason, DSM has been applied in many areas e.g. diffusive tomography [11], electrical impedance tomography [12], source detection in stratified ocean waveguide [13], etc.

In a recent study [14], the MUSIC algorithm was designed for imaging small and extended anomalies from measured $S$-parameter data; however, DSM has not yet been designed and used to identify unknown anomalies. To address this issue, we design a DSM from $S$-parameter data collected by a small number of dipole antennas to identify the outline shape anomaly with different conductivity and permittivity compared to the background medium and a significantly smaller diameter than the wavelength. To investigate the feasibility of the designed DSM, we establish a relationship between the indicator function of DSM and an infinite series of Bessel functions of integer order. Subsequently, we present the simulation results that confirm the established relationship using synthetic data generated by the CST STUDIO SUITE.

The remainder of this paper is organized as follows. In Section 2, we briefly introduce the DSM for imaging anomalies from $S$-parameter data. Subsequently, in Section 3, we present simulation results for the synthetic data generated at $f = 1$ GHz of angular frequency, which is followed by a brief conclusion in Section 4.

## 2. Preliminaries

In this section, we briefly survey the three-dimensional forward problem in which an anomaly D with a smooth boundary $\partial$D is surrounded by $N$-different dipole antennas. For simplicity, we assume that D is a small ball with radius $\rho$, which is located at $\mathbf{r}_{\mathrm{D}}$ such that

$$\mathrm{D} = \mathbf{r}_{\mathrm{D}} + \rho\mathbf{B},$$

where $\mathbf{B}$ denotes a simply connected domain. We denote $\mathbf{r}_{\mathrm{TX}}$ as the location of the transmitter, $\mathbf{r}_{\mathrm{RX}}^{(n)}$ as the location of the $n$th receiver, and $\varGamma$ as the set of receivers.

$$\varGamma = \{\mathbf{r}_{\mathrm{RX}}^{(n)} : n = 1, 2, \ldots, N \quad \text{with} \quad |\mathbf{r}_{\mathrm{RX}}^{(n)}| = R\}.$$

Throughout this paper, every material and anomaly to be non-magnetic so that they are classified on the basis of the value of their dielectric permittivity and electrical conductivity at a given angular frequency $\omega = 2\pi f$. To reflect this, we set the magnetic permeability to be constant at every location such that $\mu(\mathbf{r}) \equiv \mu = 4\cdot 10^{-7}\pi$, and we denote $\varepsilon_{\mathrm{B}}$ and $\sigma_{\mathrm{B}}$ as the background permittivity and conductivity, respectively. By analogy, $\varepsilon_{\mathrm{D}}$ and $\sigma_{\mathrm{D}}$ are respectively those of D. Then, we introduce piecewise constant permittivity $\varepsilon(\mathbf{r})$ and conductivity $\sigma(\mathbf{r})$,

$$\varepsilon(\mathbf{r}) = \begin{cases} \varepsilon_{\mathrm{D}} & \text{if } \mathbf{r} \in \mathrm{D}, \\ \varepsilon_{\mathrm{B}} & \text{if } \mathbf{r} \in \mathbb{R}^3 \setminus \overline{\mathrm{D}}, \end{cases} \quad \text{and} \quad \sigma(\mathbf{r}) = \begin{cases} \sigma_{\mathrm{D}} & \text{if } \mathbf{r} \in \mathrm{D}, \\ \sigma_{\mathrm{B}} & \text{if } \mathbf{r} \in \mathbb{R}^3 \setminus \overline{\mathrm{D}}, \end{cases}$$

respectively. Using this, we can define the background wavenumber $k$ as

$$k = \omega^2\mu\left(\varepsilon_{\mathrm{B}} + i\frac{\sigma_{\mathrm{B}}}{\omega}\right) = \frac{2\pi}{\lambda},$$

where $\lambda$ denotes the wavelength such that $\rho < \lambda/2$.

Let $\mathbf{E}_{\mathrm{inc}}(\mathbf{r}_{\mathrm{TX}}, \mathbf{r})$ be the incident electric field in a homogeneous medium due to the point current density at $\mathbf{r}_{\mathrm{TX}}$. Then, based on the Maxwell equation, $\mathbf{E}_{\mathrm{inc}}(\mathbf{r}_{\mathrm{TX}}, \mathbf{r})$ satisfies

$$\nabla \times \mathbf{E}_{\mathrm{inc}}(\mathbf{r}_{\mathrm{TX}}, \mathbf{r}) = -i\omega\mu\mathbf{H}(\mathbf{r}_{\mathrm{TX}}, \mathbf{r}) \quad \text{and} \quad \nabla \times \mathbf{H}(\mathbf{r}_{\mathrm{TX}}, \mathbf{r}) = (\sigma_{\mathrm{B}} + i\omega\varepsilon_{\mathrm{B}})\mathbf{E}_{\mathrm{inc}}(\mathbf{r}_{\mathrm{TX}}, \mathbf{r}).$$