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Existence of Infinitely Many Solutions for Fourth-Order Impulsive Differential Equations *

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Abstract: The aim of this paper is to study the existence of infinitely many solutions for fourth-order impulsive differential equations involving oscillatory behaviors of nonlinearity at infinity. The result is proved by using critical point theory and variational approach.
Keywords: Fourth-order; Oscillatory behavior; Impulsive effects; Critical point theory.
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1 Introduction

In this paper, we will study the boundary value problem of fourth-order differential equations of the type

$$\begin{cases} u^{(iv)}(t) + Au''(t) + Bu(t) = f(t, u(t)), \quad t \in [0, T]\backslash\{t_1, t_2, \cdots, t_l\}, \\ \Delta u'''(t_j) = I_{1j}(u(t_j)), \quad j = 1, 2, \cdots, l, \\ \Delta u''(t_j) = I_{2j}(u'(t_j)), \quad j = 1, 2, \cdots, l, \\ u(0) = u(T) = 0, \quad u''(0) = u''(T) = 0, \end{cases} \tag{1.1}$$

where $A \in \mathbb{R},\ B \in \mathbb{R},\ f \in C([0, T] \times \mathbb{R}; \mathbb{R}),\ I_{1j},\ I_{2j} \in C(\mathbb{R}; \mathbb{R}),\ j = 1, 2, \cdots, l,\ 0 = t_0 < t_1 < \cdots < t_l < t_{l+1} = T,\ \Delta u'''(t_j) = u'''(t_j^+) - u'''(t_j^-),\ \Delta u''(t_j) = u''(t_j^+) - u''(t_j^-)$ with $u'''(t_j^{\pm}) = \lim_{t\to t_j^{\pm}} u'''(t)$ and $u''(t_j^{\pm}) = \lim_{t\to t_j^{\pm}} u''(t)$.

In the past few years, much work has been done in the study of the existence of solutions for boundary value problems with impulses, by which a number of physical situations, biological phenomena, population dynamics, optimal control, industrial robotic and chemotherapy are described. We refer the readers to the classical monographs [10, 11, 12, 16].

Recently, a great deal of papers have worked on fourth-order differential equations (see [3, 4, 5, 11, 13, 14, 17]). Bonanno and Bella [14] have studied the following problem

$$\begin{cases} u^{(iv)}(t) + Au''(t) + Bu(t) = \lambda f(t, u), \text{ in } [0, 1], \\ u(0) = u(1) = 0, \\ u''(0) = u''(1) = 0. \end{cases}$$

The authors presented multiplicity results based on the critical point theory. Sun and Chen [13] studied the problem

$$\begin{cases} u^{(iv)}(t) + Au''(t) + Bu(t) = f(t, u(t)), \quad a.e.\ t \in [0, T], \\ \Delta u''(t_j) = I_{1i}(u'(t_j)), \quad i = 1, 2, \cdots, l, \\ \Delta u'''(t_j) = I_{2i}(u(t_j)), \quad i = 1, 2, \cdots, l, \\ u(0) = u(T) = 0, \quad u''(0) = u''(T) = 0. \end{cases}$$

By variational methods and critical point theory, they gave some new criteria to guarantee that the impulsive problem has at least one nontrivial solution, infinitely many distinct solutions under some different conditions. By critical point theory, Tian and Liu [17] have obtained existence results of solutions for the following problem

$$\begin{cases} u^{(iv)}(t) - u''(t) + u(t) = f(t, u(t)), \quad t \in [0, T]\backslash\{t_1, t_2, \cdots, t_l\}, \\ -\Delta u'''(t_i) = I_{1i}(u(t_i)), \quad i = 1, 2, \cdots, l, \\ -\Delta u''(t_i) = I_{2i}(u'(t_i)), \quad i = 1, 2, \cdots, l, \\ au(0) - bu'(0) = 0,\ cu(T) + du'(T) = 0, \\ au''(0) - bu'''(0) = 0,\ cu''(T) + du'''(T) = 0. \end{cases}$$

Fourth-order impulsive differential equations were also studied by topological degree theory and cone theory (see [3, 4, 5]). However, to our best knowledge, there are few papers studying the existence of infinitely many solutions for fourth-order impulsive boundary value problem. In this paper, we will obtain the existence of infinitely many solutions for problem (1.1) with oscillatory behaviors of nonlinearity at infinity by using critical point theory.

The rest of this paper is organized as follows. In Section 2, we present several preliminaries. In Section 3, we prove the main result. In Section 4, we give an example to illustrate the main result.

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