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A new stability test for linear neutral differential equations

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Abstract

We obtain new explicit exponential stability conditions for the linear scalar neutral equation with two bounded delays $\dot{x}(t) - a(t)\dot{x}(g(t)) + b(t)x(h(t)) = 0$, where $0 \le a(t) \le A_0 < 1$, $0 < b_0 \le b(t) \le B$, using the Bohl-Perron theorem and a transformation of the neutral equation into a differential equation with an infinite number of delays. The results are applied to the neutral logistic equation.

Keywords: neutral equations, uniform exponential stability, Bohl-Perron theorem, variable delays, explicit stability conditions, logistic neutral differential equation
AMS Subject Classification: 34K40, 34K20, 34K06

1. Introduction

Many applied problems lead to neutral differential equations as their mathematical models, for example, a model of a controlled motion of a rigid body, a distributed network (a long line with tunnel diodes), models of infection diseases, a price model in economic dynamics, see, for example, [3, 14, 15]. Though neutral delay differential equations describe important applied models, from mechanics to disease spread in epidemiology, compared to other classes of equations, stability theory for neutral equations with variable coefficients and delays is not sufficiently developed. In particular, there are no explicit stability results for general linear equations but only for particular classes of neutral equations, see [7, 8, 10, 11, 16] and references therein.

The aim of the present paper is to obtain stability conditions for the equation

$$\dot{x}(t) - a(t)\dot{x}(g(t)) = -b(t)x(h(t)) \qquad (1.1)$$

which depend on both delays. To this end, we transform (1.1) into a linear delay differential equation with an infinite number of delays. This method has not been applied before to stability problems, but used to study oscillation in [4].

As an application, we give local asymptotic stability tests for the logistic neutral equation

$$\dot{x}(t) = r(t)x(t)\left(1 - \frac{x(h(t)) - \rho\dot{x}(g(t))}{K}\right), \qquad (1.2)$$

where $\rho > 0$ corresponds to higher resources consumption by a shrinking population. The model

$$\dot{x}(t) = r_0 x(t)\left(1 - \frac{x(t-\tau) - \rho\dot{x}(t-\tau)}{K}\right) \qquad (1.3)$$

which is an autonomous version of (1.2), was studied in [9, 13, 17].

2. Preliminaries

We consider scalar delay differential equation (1.1) under the following conditions:
(a1) a, b, g, h are Lebesgue measurable, a and b are essentially bounded on $[0, \infty)$ functions;