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Semi-rational solutions for the (2 + 1)-dimensional nonlocal Fokas system

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ABSTRACT

The $(2+1)$-dimensional $[(2+1)d]$ Fokas system is a natural and simple extension of the nonlinear Schrödinger equation (see Eq. (2) in Fokas, 1994). In this letter, we introduce its $\mathcal{PT}$-symmetric version, which is called the $(2+1)d$ nonlocal Fokas system. The N-soliton solutions for this system are obtained by using the Hirota bilinear method whereas the semi-rational solutions are generated by taking the long-wave limit of a part of exponential functions in the general expression of the N-soliton solution. Three kinds of semi-rational solutions, namely (1) a hybrid of rogue waves and periodic line waves, (2) a hybrid of lump and breather solutions, and (3) a hybrid of lump, breather, and periodic line waves are put forward and their rather complicated dynamics is revealed.

1. Introduction

The study of nonlinear evolution equations (NLEEs) has made fantastic progress, and it is frequently used to model various nonlinear phenomena in physics, chemistry, biology, and even social sciences. The analytical solutions of NLEEs are necessary for a better understanding of those nonlinear phenomena. Many researchers have made great efforts to find analytical solutions of those equations and a series of powerful techniques have been proposed, such as the Lie group analysis [1,2], the inverse scattering transformation (IST) [3,4], the Darboux transformation (DT) [5,6], the Hirota bilinear method [7–9], and other techniques [10–15]. To date, most of the relevant studies were focused on solitons, breathers, rogue waves (RWs), and lump solutions. However, during the past few years, studies of the dynamics of semi-rational solutions of NLEEs have become an important research field [16–19]. Generally speaking, comparing to $(1+1)$-dimensional $[(1+1)d]$ systems, the methods to solve $(2+1)$-dimensional $[(2+1)d]$ systems are much more complicated.

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Therefore, the study of $(2+1)d$ semi-rational solutions of NLEEs would be much more challenging and meaningful.

Parity-time $[\mathcal{PT}]$ symmetry of physical systems has been extensively studied both theoretically and experimentally since the pioneering works of Bender et al. [20–22], who showed that a wide class of non-Hermitian Hamiltonians having the $\mathcal{PT}$ symmetry can possess entirely real spectra as long as this symmetry is not spontaneously broken. The $\mathcal{PT}$ symmetry is maintained in optics by means of a special balance between gain and loss in the corresponding regions of the optical system [23,24]. The studies of $\mathcal{PT}$ systems have led to new developments in diverse areas of theoretical and applied physics, including quantum field theories [25], Lie algebras [26], complex crystals [27–29], and optics and photonics [30].

Inspired by the above results of $\mathcal{PT}$-symmetric physical systems, we introduce a new $\mathcal{PT}$-symmetric equation,

$$iU(x,y,t)_t + U(x,y,t)_{xx} + U(x,y,t)\int_{-\infty}^{y}[U(x,y,t)U(-x,-y,t)^*]_x dy' = 0, \tag{1}$$

which is an extension of the $(2+1)d$ Fokas system [31], and thus is called the $(2+1)d$ nonlocal Fokas system. There are some interesting results on the $(2+1)d$ Fokas system [32–36], including solitons, lumps, and line-rogue waves. Instead, we shall focus on the above newly established nonlocal system, and construct its N-soliton and semi-rational solutions.

Using the transformations

$$V_y = [U(x,y,t)U^*(-x,-y,t)]_x. \tag{2}$$

Eq. (1) becomes the following system of coupled partial differential equations:

$$\begin{aligned} &iU_t + U_{xx} + UV = 0,\\ &V_y = [U(x,y,t)U^*(-x,-y,t)]_x, \end{aligned} \tag{3}$$

where U and V are two real functions that satisfy the two-dimensional [(2d)] $\mathcal{PT}$-symmetry condition $V(x,y,t) = V^*(-x,-y,t)$.

The organization of this paper is as follows. In Section 2, N-soliton solutions are derived by employing the Hirota bilinear method. In Section 3, three kinds of semi-rational solutions are generated by taking the long wave limit of a part of exponential functions in the general expression of the N-soliton solution obtained with Hirota method. The main results of the paper are summarized in Section 4.

2. N-soliton solution of the $(2+1)d$ nonlocal Fokas system

The bilinear forms of Eq. (1) have been given [36]

$$\begin{aligned} &(D_x^2 + iD_t)g\cdot f = 0,\\ &(D_xD_y + 1)f\cdot f = gg^*, \end{aligned} \tag{4}$$

through the dependent variable transformation:

$$U = g/f, \qquad V = 2(log f)_{xx}, \tag{5}$$

where D is the Hirota's bilinear differential operator, and f and g are functions of x, y, and t, subject to the condition:

$$f(x,y,t) = f^*(-x,-y,t). \tag{6}$$