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Using inverse Lz-transform for obtaining compact stochastic model of complex power station for short-term risk evaluation

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ABSTRACT

In this paper a short-term risk evaluation is performed for electric power stations, where each power generating unit is presented by a multi-state Markov model. Risk is treated as the probability of loss of load or in other words, as the probability of system entrance in the set of states, where demand cannot be satisfied. The main obstacle for risk evaluation in such cases is a "curse of dimensionality" – a great number of states of entire power station that should be analyzed. Usually when the number of system states is increasing drastically, enormous efforts are required for solving the problem by using classical Markov methods or simulation techniques. Well known universal generating function technique also cannot be directly applied because this technique is primarily oriented to steady-state reliability analysis. By using such extension of UGF techniques as L_z-transform, one can find for the short-term such reliability characteristics as loss of load probability, expected energy not supplied to consumers etc. However risk function still cannot be obtained by using these techniques. This problem in many practical cases is a challenge for reliability researchers and engineers. In this paper, a special method based on inverse L_z-transform (L_Z^{-1} transform) is developed in order to calculate risk function for such multi-state power systems. In order to illustrate the proposed approach, the short-term risk evaluation for a power station with different coal-fired generating units is presented.

1. Introduction

Multi-state system (MSS) reliability analysis and optimization is one of most intensively developing areas in modern reliability theory [1,2,17]. Multi-state models are widely used in the field of power system reliability assessment [3]. It has been recognized [4] that using simple two-state models for large generating units in generating capacity adequacy assessment can yield pessimistic appraisals. In order to more accurately assess power system reliability, many electric utilities now utilize multi-state models instead of two-state representations [4,5]. A technique, called the apportioning method [4], is usually used to create steady-state multi-state generating unit models based on real world statistical data for generating units. Utilizing this technique, steady-state probabilities of generating units residing at different generating capacity levels can be defined. When the short-term behavior of a MSS is studied, the investigation cannot be based on steady-state (long-term) probabilities. This investigation should use the MSS model, in which transition intensities between any states of the model are known. Such general multi-state Markov model was considered for coal fired generating unit in [6]. This paper describes the method for transition intensities estimation from actual generating unit failures (deratings) and repair statistics, which is presented by the observed realization of generating capacity stochastic process.

It was shown that such important reliability indices as Loss of Load Probability (LOLP), Expected Energy Not Supplied (EENS) to consumers, etc., which were found for a short time, are essentially different from those found for a long-term evaluation. Especially important is a fact that these indices strongly depend on power system initial condition. Usually in each power station there are a number of generating units. If a power station consists of n generating units where each unit is represented by m-state Markov model, then the Markov model for the entire power station will have m^n states. In order to find reliability indices for entire power station, this complicated model should be built and analyzed. As can be seen, it will require enormous efforts even for relatively small m and n. In order to overcome this obstacle, a specific approach called the Universal Generating Function (UGF) technique has been introduced in [7] and then successfully applied to MSS reliability analysis [8,9] and to power system reliability analysis and optimization [21–23]. The main limiting factor of the UGF technique application to power system reliability analysis is that UGF is based on a moment generating function that is mathematically defined only for *random variable*. This fact is a reason for

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considering a performance (capacity) of each system component as a random variable, in spite of the fact that in reality it is a discrete-state continuous-time stochastic process [2]. In practice this important restriction leads one to consider only steady-state parameters of power system. Monte Carlo simulation technique is another technique for reliability evaluation of MSS, which is used for reliability assessment of restructured power systems [19]. However Monte Carlo simulation technique for reliability assessment of large scale MSS is time consuming, which restricts its practical application. In order to extend UGF technique application to dynamic reliability models a special mathematical technique –L_Z-transform was suggested in [10]. Recently there are some successful applications of L_Z-transform method to dynamic reliability analysis of general MSS [11,12].

A L_Z-transform technique has been demonstrated in [13] for power system short-term evaluation for such important indices as availability, expected energy not supplied, expected capacity deficiency. However for effective power system dispatch it is often important to obtain power system risk function, which cannot be found by using L_Z-transform.

For example, it may be needed to know how much time a system has under specified initial conditions up to its entrance in the failure state, when required power demand will not be satisfied. Therefore it is important to know probability distribution of time up to the failure, where failure is treated as the system entrance in the set of states with unsatisfied demand. In other words, evaluation of power system risk function $\text{Risk}(t)$ is necessary.

Based on the risk function the system operator can make appropriate operating decisions such as starting reserve generators, unit shut down in order to provide maintenance and so on. For these purposes, this paper suggested an approach that is based on inverse L_Z-transform (L_Z^{-1} – transform)that was introduced and mathematically defined in [14]. Based on this approach a power system risk function can be found and system operator can estimate risk corresponding to each operating decision.

Generally the suggested approach can be presented by the following steps:

1. Solving classical differential equations for Markov model of each system element in order to obtain state probabilities as functions of time and determine *Lz*-transform for each individual element.
2. Obtaining resulting *Lz*-transform for the entire system output Markov process by using Ushakov's universal generating operator and corresponding UGF techniques.
3. Uncovering underlying Markov process for the obtained resulting *Lz*-transform by utilizing inverse *Lz*-transform.
4. Investigating uncovered Markov process for obtaining the risk function of the entire system.

A numerical example illustrates the application of the approach to power system risk analysis and corresponding benefits.

2. Inverse L_Z-transform: definitions and computational procedures

2.1. Definitions

Here we present a brief description of L_Z-transform and inverse L_Z-transform.

Consider a discrete-state continuous-time (DSCT) Markov process [15], $X(t) \in \{x_1, \dots, x_K\}$, $t \geq 0$, which has K possible states x_i, $i=1,\dots,K$. In MSS reliability theory $x_1,\dots, x_K$ usually are interpreted as possible performance levels. This Markov process is completely defined by set of K possible states $\mathbf{x} = \{x_1, \dots, x_K\}$, transitions intensities matrix $\mathbf{A} = \|a_{ij}(t)\|$, $i, j = 1, \dots, K$ and by initial states probability distribution that can be presented by the corresponding set:

$$\mathbf{p_0} = \left[p_{10} = \Pr\{X(0) = x_1\}, \dots, p_{K0} = \Pr\{X(0) = x_K\} \right].$$

From now on, we shall use for such Markov process the following notation by using triplet:

$$X(t) = \left\{ \mathbf{x}, \mathbf{A}, \mathbf{p_0} \right\}. \tag{1}$$

Remark. If functions $a_{ij}(t) = a_{ij}$ are constants, then the DSCT Markov process is said to be *time-homogeneous*. When $a_{ij}(t)$are time dependent, then the resulting Markov process is non-homogeneous.

Definition 1. [10]. L_Z-transform of a discrete-state continuous-time Markov process $X(t) = \{\mathbf{x}, \mathbf{A}, \mathbf{p_0}\}$ is a function $u(z, t, \mathbf{p_0})$ defined as

$$L_Z\{X(t)\} = u(z, t, \mathbf{p_0}) = \sum_{i=1}^{K} p_i(t) z^{x_i}. \tag{2}$$

where $p_i(t)$ is the probability that the process is in state i at time instant $t \geq 0$ *f*or any given initial states probability distribution $\mathbf{p_0}$, and z in general is a complex variable.

It was proven in [10] that for discrete state continuous time Markov process, $X(t) = \{\mathbf{x}, \mathbf{A}, \mathbf{p_0}\}$, where transition intensities $a_{ij}(t)$ are *continuous functions of time*, exists one and only one (unique) L_Z-transform. In other words, each discrete-state continuous-time Markov process *with continuous transition intensities* under certain initial conditions $\mathbf{p_0}$ has only one (unique) L_Z-transform $u(z, t, \mathbf{p_0})$ and each L_Z-transform $u(z, t, \mathbf{p_0})$ will have only one corresponding DSCT Markov process $X(t)$ developing from these initial conditions.

Remark. The main reason for *Lz*-transform introduction in [10] was the fact that UGF is defined only for random variables, not for stochastic processes. Therefore states probabilities p_i in the UGF expression are not dependent on time *t*. However, in practice often it is not enough to present MSS and its components as random variables. Generally, when it is needed to investigate transient modes, aging processes, etc., the system and its components should be described as stochastic processes. Therefore, by using UGF it was possible to investigate only steady-state behavior of MSS. Transient modes, aging processes, etc. where states probabilities p_i(t) are time-dependent (functions of time *t*) were out of the scope of UGF. As it was proven in [10], based on *Lz*-transform it is possible to utilize Ushakov's universal generating operator in order to perform MSS reliability analysis in all these modes by using well established technique of Ushakov's operator. This technique is often called as UGF technique. So, *Lz*-transform is a new mathematical object that extends application of this technique to MSS, where its components are described by using stochastic (discrete-state continuous-time Markov) processes. The unique condition that should be fulfilled is the following: all Markov stochastic processes under consideration should have transition intensities $a_{ij}(t)$ that are continuous functions of time.

Definition 2. [14]. *Inverse* L_Z*- transform* (L_Z^{-1}*-transform*) *of a function* (2) where each $p_i(t)$is a probability that some ***unknown*** discrete-state continuous-time Markov process $X(t)$ (which begins under certain initial conditions $\mathbf{p_0}$at instant $t = 0$) is in state *i* at time instant $t \geq 0$, x_i is the performance in this state, and z is a complex variable,