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Analysis on dynamics of a population model with predator–prey-dependent functional response☆

Yunfeng Jia

School of Mathematics and Information Science, Shaanxi Normal University, Xi'an, Shaanxi 710062, China

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ABSTRACT

We consider a reaction–diffusion population model with predator–prey-dependent functional response. Firstly, we discuss the conditions which ensure the model has a unique positive constant solution. Secondly, we investigate the dynamical properties of the model, including the large time behaviors of the nonconstant solutions and the local and the global asymptotic stability of the positive constant solution.

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1. Introduction

The predator–prey type model is a very important branch in numerous ecological–mathematical models. One of the important themes for both ecological and mathematical models is the dynamical relationship between predators and their prey. In [1], the authors pointed out that the prey-dependent type models have been challenged since the fact that the rate of the predator consuming prey does not only depend on prey density but also depends on the density of predator itself. In fact, in many cases, especially when the predators search for food, they always have to share food when they compete for food. Consequently, some scholars think that the functional response in predator–prey models should be predator–prey-dependent. As a result, some predator–prey-dependent type models with different functional responses (such as Hassell–Varley type [2,3], Beddington–DeAngelis type [1,4,5] and Crowley–Martin type [6,7] functional responses) emerged successively.

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E-mail address: jiayf@snnu.edu.cn.

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In [8], the authors studied the following predator–prey-dependent model with non-selective harvesting

$$\begin{cases} u_t = ru(1-\dfrac{u}{k}) - \dfrac{\alpha uv}{a+bu+cv} - q_1 meu, \\ v_t = sv(1-\dfrac{v}{l}) + \dfrac{\beta uv}{a+bu+cv} - q_2 mev, \end{cases} \tag{1.1}$$

where u and v are two fish species, and represent densities of prey and predator, respectively; r and s are the intrinsic growth rate of u and v; α is the maximal relative increase of predation, and β represents the conversion efficiency of consumed prey into new predators; a, b and c describe the effects of capture rate, the handling time on the feeding rate and the magnitude of interference among predators, respectively; $q_i, i = 1, 2$ are the catchability coefficients, e is the combined fishing effort used to harvest the population, and m is the fraction of the stock available for harvesting. All parameters are assumed to be positive.

In order to consider the natural tendency of each species to diffuse to areas of smaller population concentration and the inhomogeneous distribution of the predator and prey in different spatial location within a fixed bounded domain at any given time, the corresponding PDE system with Dirichlet boundary conditions of (1.1) is analyzed in [9]. In this paper, we focus on the following PDE system of reaction–diffusion type corresponding to (1.1) with Neumann boundary conditions and without non-selective harvesting effort

$$\begin{cases} u_t - \Delta u = ru(1-\dfrac{u}{k}) - \dfrac{\alpha uv}{a+bu+cv}, & (x,t) \in \Omega \times (0,\infty), \\ v_t - \Delta v = sv(1-\dfrac{v}{l}) + \dfrac{\beta uv}{a+bu+cv}, & (x,t) \in \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0, & (x,t) \in \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) \geq, \not\equiv 0,\ v(x,0) = v_0(x) \geq, \not\equiv 0,\ x \in \Omega, \end{cases} \tag{1.2}$$

where $\Omega \subset R^N$ is an open and bounded domain with smooth boundary $\partial\Omega$, $\frac{\partial}{\partial n}$ is the outward unit normal vector on $\partial\Omega$, $u_0(x)$ and $v_0(x)$ are continuous functions in $\overline{\Omega}$. The boundary conditions indicate that there has no populations flux across $\partial\Omega$ for two species.

2. Main results

We give some dynamical behavior analysis on the system (1.2). Firstly, the sufficient conditions are presented to ensure the existence and uniqueness of the positive constant solution to the system. Denote by (u^*, v^*) the constant solution. Then (u^*, v^*) satisfies

$$\begin{cases} r(1-\dfrac{u^*}{k}) - \dfrac{\alpha v^*}{a+bu^*+cv^*} = 0, \\ s(1-\dfrac{v^*}{l}) + \dfrac{\beta u^*}{a+bu^*+cv^*} = 0. \end{cases} \tag{2.1}$$

Through the first equation of (2.1), we obtain

$$v^* = \frac{r(a+bu^*)(1-\frac{u^*}{k})}{\alpha - cr(1-\frac{u^*}{k})}. \tag{2.2}$$

Substitute (2.2) into the second equation of (2.1) to get a cubic equation on u^*

$$A(u^*)^3 + B(u^*)^2 + Cu^* + D = 0 \tag{2.3}$$

with

$$A = \frac{rs\alpha b^2}{k} + \frac{l\beta c^2 r^2}{k^2} > 0, \quad B = \frac{bcrsl\alpha}{k} + \frac{2abrs\alpha}{k} - r(sb^2\alpha + \frac{2lr\beta c^2}{k}) + \frac{2clr\alpha\beta}{k},$$
$$C = s\alpha(\frac{aclr}{k} + \frac{ra^2}{k} - br(2a+cl) + bl\alpha) + l\beta(\alpha - cr)^2, \quad D = as\alpha(l\alpha - clr - ar).$$

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