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Nonparametric estimation in trend-renewal processes

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ABSTRACT

The trend-renewal-process (TRP) is defined to be a time-transformed renewal process, where the time transformation is given by a trend function $\lambda(\cdot)$ which is similar to the intensity of a nonhomogeneous Poisson process (NHPP). A nonparametric maximum likelihood estimator of the trend function of a TRP can be obtained in principle in a similar manner as for the NHPP using kernel smoothing. For a full nonparametric estimation of a trend-renewal process it is necessary, however, to estimate jointly the trend function and the renewal distribution. For this purpose we consider a nonparametric approach using kernel smoothing techniques. We develop an original algorithm to estimate the conditional intensity function by preserving its structure in terms of the trend function and the underlying renewal process. The algorithm is applied to both simulated and real data sets.

1. Introduction

Repairable systems are systems that are repaired to satisfactory performance after a failure. Traditionally, the literature on repairable systems treats the failure times using point process theory [2,25]. The most common models for the failure process of a repairable system are nonhomogeneous Poisson processes (NHPP), related to what is called minimal repairs, and renewal processes (RP), related to perfect repairs or replacements. For many applications it is more reasonable, however, to model the repair action by something in between the two given extremes, leading to the need for more general models, often called imperfect repair models.

The classical imperfect repair model is the one suggested by Brown and Proschan [7], which assumes that at the time of each failure a perfect repair occurs with probability p and a minimal repair occurs with probability $1-p$, independently of the previous failure history. This model has been generalized in several directions, many of them using the concept of *effective age* or *virtual age*. The most famous of the latter type of models was suggested by Kijima [19]. The idea is to distinguish between the system's true age, which is the time elapsed since the system was new, and the virtual age of the system which describes its present condition when compared to a new system. The virtual age is redefined at failures according to the type of repair performed, and runs along with the calendar time between failures. Doyen and Gaudoin [10] studied several classes of virtual age models based on deterministic reduction of effective age due to repairs. In a recent review article, Tanwar et al. [28] conduct a thorough survey of virtual age models described in terms of the so-called Generalized Renewal Process (GRP) introduced in [20].

Imperfect repair models are often the basis for optimal maintenance policies. For example, Yevkin and Krivtsov [31] use the GRP in an optimal preventive maintenance problem. Another generalization of the renewal process, the quasi-renewal process or geometric renewal process, is used in an optimal maintenance problem by Lam [21].

The trend-renewal-process (TRP), described and studied in Lindqvist, Elvebakk and Heggland [22], is a different kind of imperfect repair model. The TRP is defined to be a time-transformed renewal process, where the time transformation is given by a trend function, $\lambda(\cdot)$, similar to the intensity of a non-homogeneous Poisson process (NHPP), and the renewal process is characterized by the interarrival distribution, F. In some sense the TRP is constructed as the "least common multiple" of the NHPP and the RP. Thus the TRP framework can be used to distinguish between the two "extreme" kinds of repair, minimal and perfect. In addition, the TRP is able to represent a possible trend in interfailure times.

It is this ability to incorporate two very different features of a repairable system, the type of repair and a possible time trend in failure occurrences, that makes the TRP a powerful tool, despite its simple structure.

The present paper is concerned with the fitting of TRP models to failure data. Until now, this has been mostly done using

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parametric models for the trend function and the underlying renewal process, see, e.g., Lindqvist et al. [22], Cook and Lawless [9, Ch. 5.2], Bebbington [5], Jokiel-Rokita and Magiera [17], and Franz, Jokile-Rokita and Magiera [11], for examples.

However, in many applications there is no clear reason to choose a concrete parametric model for the conditional intensity function and in such a case a free-model method and then a data-driven focus of the problem seems to be the most adequate.

Heggland and Lindqvist [16] considered nonparametric estimation of an assumed monotone trend function $\lambda(\cdot)$ when F is parametric, in particular Weibull distributed. Later, Lindqvist [23] considered the case when $\lambda(\cdot)$ is a general nonnegative function, thus extending earlier work on nonparametric estimation in NHPPs, e.g., Bartozinski et al. [4]. A review of these studies can be found in the monograph by Gamiz, Kulasekera, Limnios and Lindqvist [13].

In the present paper we consider a fully nonparametric approach and estimate the conditional intensity function of the TRP by using kernel smoothing techniques. We develop an algorithm to estimate the conditional intensity function by preserving its structure in terms of the trend function and the underlying renewal process, obtaining a weighted kernel estimator for the trend function motivated by the approach of Ramlau-Hansen [27], while proceeding similar to Nielsen and Tanggaard [26] for the estimation of the hazard of the renewal distribution. Thus in the following we do not assume any particular functional form for the distribution function of the underlying RP, F, and the trend function $\lambda(\cdot)$.

Although it still seems that parametric models are the first choice in reliability analyses, there is an increasing activity in the use of nonparametric methods. For example, Gamiz and Roman [12]; Xiao and Dohi [30] and Taylor and Pena [29] use nonparametric methods in applications related to repairable systems, while Gandy and Jensen [15]; Luo et al. [24]; Bobrowski et al. [6]; Zhao et al. [32] use kernel smoothing methods in specific reliability applications.

The present paper is organized as follows. Section 2 states the basic notation and definitions of the point processes used for modeling of repairable systems, in particular the TRP. In addition the section gives a brief presentation of kernel estimation methods to be used in the algorithm of the paper. The algorithm for nonparametric estimation in TRPs is presented in Section 3 to be considered in the rest of the paper. Then in Section 4 it is performed a simulation study. Section 5 presents applications of the algorithm to three different data sets which have previously appeared in the literature. It is indicated how our results are in accordance with results from earlier studies of these data. Some concluding remarks are finally given in the final section of the paper.

2. Preliminaries

2.1. Notation

Consider a repairable system, observed from time $t=0$. To our concern, the time-behavior of the system is represented by a counting process $\mathbf{N} = \{N(t), t \geq 0\}$, where $N(t)$ counts the number of failures of the system observed in $(0, t]$. Let T_i be the time of the ith failure, where we define $T_0 = 0$, and let X_i be the time between failure number $i-1$ and failure number i, that is $X_i = T_i - T_{i-1}$. We assume that all repair times equal 0. This assumption is reasonable if the repair times are negligible compared to the times between failures, or if we let the time parameter be the operation time of the system. For a general treatment of repairable systems, see the previously cited monographs [2,25].

The counting process $\mathbf{N}$ can be completely characterized by its conditional intensity function which is defined as follows.

2.2. Conditional intensity function

Let $\mathcal{F}_{t^-}$ denote the history of the process up to, but not including, time t. $\mathcal{F}_{t^-}$ is the sigma-algebra generated by the set $\{N(s), 0 \leq s < t\}$ and hence it contains all the information about failure times in the past, that is until t, and is called a *filtration*. The conditional intensity function is defined as

$$\gamma(t) = \lim_{h \to 0} \frac{P\{N(t+h) - N(t) > 0 | \mathcal{F}_{t^-}\}}{h}. \tag{1}$$

2.3. Models for repairable systems

In the following we review the definitions of the RP and NHPP, and then we define the trend renewal process (TRP) which will be the main model used in this paper and which can be seen as a generalization of the first two models, see [13].

The process $N(t)$ is a *renewal process* with interarrival distribution F, RP(F), if $X_1, X_2, \ldots$ are independent and identically distributed with cumulative distribution function (cdf) F, where we assume that $F(0) = 0$. If F is the exponential distribution with parameter λ, then RP(F)=HPP(λ), the homogeneous Poisson process with intensity λ. If we denote by z the hazard function corresponding to one of the X_i, the conditional intensity function of a renewal process defined in (1) is given by

$$\gamma(t) = z(t - T_{N(t^-)}). \tag{2}$$

Let $\lambda(t)$, $t \geq 0$ be a nonnegative function and let the corresponding cumulative function be $\Lambda(t) = \int_0^t \lambda(s)\, ds$. The process $N(t)$ is a nonhomogeneous Poisson process with intensity function $\lambda(t)$, NHPP ($\lambda(\cdot)$), if the time-transformed process $\Lambda(T_1), \Lambda(T_2), \ldots$ is an HPP (1). In this case, the conditional intensity function defined in (1) does not depend on the history of the process, and we have that $\gamma(t) = \lambda(t)$.

The *trend-renewal process*, TRP(F, $\lambda(\cdot)$), combines the two definitions above as follows. Let $\lambda(t)$ and $\Lambda(t)$ be as for the NHPP, and let F be a cdf with $F(0) = 0$. Then the process $N(t)$ is a TRP(F, $\lambda(\cdot)$) if the time-transformed process $\Lambda(T_1), \Lambda(T_2), \ldots$ is an RP(F). The function F represents the *renewal distribution*, and $\lambda(\cdot)$ is called the *trend function* of the TRP. It is shown in [22] that in this case, the conditional intensity function (1) is obtained as

$$\gamma(t) = z(\Lambda(t) - \Lambda(T_{N(t^-)}))\lambda(t), \tag{3}$$

where, as for the RP, $z(\cdot)$ is the hazard rate function corresponding to F. The function z will in the following be called the *renewal hazard*.

It is easy to see that the TRP generalizes both the NHPP and the RP, since TRP($1 - e^{-x}$, $\lambda(\cdot)$)=NHPP($\lambda(\cdot)$) and TRP(F,1)=RP(F).

2.4. Kernel estimation of the intensity function $\lambda(t)$ of an NHPP

Suppose first that $T_1, T_2, \ldots, T_{N(\tau)}$ are the observed failure times of an NHPP observed on the time interval $[0, \tau]$. The standard *kernel estimator* of $\lambda(t)$ is then given by

$$\hat{\lambda}(t) = \frac{1}{b} \sum_{i=1}^{N(\tau)} K\left(\frac{t - T_i}{b}\right), \quad t > 0. \tag{4}$$

Here K denotes a kernel function, usually a symmetric density function with a compact support, while b is a bandwidth parameter conveniently chosen (see, e.g., [13, Chapter 3)].

The kernel estimator (4) may however incur an important bias when estimating $\lambda(t)$ near the boundary $t=0$. To alleviate to some