Applied Mathematics Letters 80 (2018) 79–87

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Infinitely many solutions for a class of superlinear Dirac–Poisson system[☆]

Wen Zhang[a,c], Jian Zhang[a,c,*], Weijin Jiang[b,c]

[a] School of Mathematics and Statistics, Hunan University of Commerce, Changsha, 410205 Hunan, PR China
[b] School of Computer and Information Engineering, Hunan University of Commerce, Changsha, 410205 Hunan, PR China
[c] Key Laboratory of Hunan Province for Mobile Business Intelligence, Hunan University of Commerce, Changsha, 410205 Hunan, PR China

ARTICLE INFO

Article history:
Received 15 December 2017
Received in revised form 16 January 2018
Accepted 16 January 2018
Available online 6 February 2018

Keywords:
Dirac–Poisson system
Superlinear
Variational methods
Strongly indefinite functionals

ABSTRACT

This paper is concerned with the nonlinear Dirac–Poisson system

$$\begin{cases} -i\sum_{k=1}^{3}\alpha_k\partial_k u + (V(x)+a)\beta u + \omega u - \phi u = F_u(x,u), \\ -\Delta\phi = 4\pi|u|^2, \end{cases} \quad \text{in } \mathbb{R}^3$$

where V is an external potential and F is a superlinear nonlinearity modeling various types of interactions. Existence and multiplicity of stationary solutions are obtained for the system with periodicity condition via variational methods.

1. Introduction and main result

We consider the following nonlinear Dirac–Maxwell system

$$\begin{cases} i\hbar\partial_t\psi = \sum_{k=1}^{3}\alpha_k(-ic\hbar\partial_k + A_k)\psi + mc^2\beta\psi - A_0\psi, \\ \partial_t A_0 + \sum_{k=1}^{3}\partial_k A_k = 0, \quad \partial_t^2 A_0 - \Delta A_0 = 4\pi|\psi|^2, \\ \partial_t^2 A_k - \Delta A_k = 4\pi(\alpha_k\psi)\bar{\psi}, \quad k=1,2,3, \end{cases} \quad \text{in } \mathbb{R}\times\mathbb{R}^3 \tag{1.1}$$

☆ This work was supported by the NNSF (Nos. 11701173, 11601145, 61472136, 61772196), by the Natural Science Foundation of Hunan Province (Nos. 2017JJ3130, 2017JJ3131), by the Excellent youth project of Education Department of Hunan Province (17B143), and by the Hunan University of Commerce Innovation Driven Project for Young Teacher (16QD008).
* Corresponding author at: School of Mathematics and Statistics, Hunan University of Commerce, Changsha, 410205 Hunan, PR China.
E-mail addresses: zwmath2011@163.com (W. Zhang), zhangjian433130@163.com (J. Zhang).

https://doi.org/10.1016/j.aml.2018.01.009
0893-9659/

where $\partial_k = \frac{\partial}{\partial x_k}$, $\psi(t,x) \in \mathbb{C}^4$, c is the speed of light, m is the mass of the electron, $\hbar$ is the Planck's constant, $\mathbf{A} := (A_1, A_2, A_3) : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{R}^3$ is the magnetic field, $A_0 : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{R}$ is the electric field, and $u\bar{v}$ denotes the usual scalar product of $u, v \in \mathbb{C}^4$. Furthermore, $\alpha_1, \alpha_2, \alpha_3$ and β are the 4×4 complex matrices:

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad k = 1,2,3,$$

with

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The Dirac–Maxwell system, which describes the interaction of a particle with its self-generated electromagnetic field, plays an important role in quantum electrodynamics. Also it has been used as effective theories in atomic, nuclear and gravitational physics (see [1]).

In this paper, we consider the electrostatic case, namely

$$A_0 = \phi(x), \quad A_k = 0, \ k = 1,2,3, \quad x \in \mathbb{R}^3,$$

and for standing wave function

$$\psi(t,x) = u(x)e^{i\theta t/\hbar}, \quad \theta \in \mathbb{R}, \quad u : \mathbb{R}^3 \to \mathbb{C}^4.$$

In the case of zero magnetic field (i.e. $A_k = 0, k = 1,2,3$) and non-trivial electric potential $\phi(x)$, the Dirac–Maxwell system (1.1) has the form

$$\begin{cases} -i\sum_{k=1}^{3}\alpha_k\partial_k u + a\beta u + \omega u - \phi u = 0, \\ -\Delta\phi = 4\pi|u|^2, \end{cases} \quad \text{in } \mathbb{R}^3 \tag{1.2}$$

where $a = mc/\hbar$, $\omega = \theta/c\hbar$. In [1], this system is called the Dirac–Poisson system.

When $\phi \equiv 0$, there have been some works focused on existence of nontrivial solution, multiple solutions and ground state solutions for system (1.2) and its variants by using various variational techniques. See for example, [2–7] and the references therein.

When $\phi \neq 0$, system (1.2) is a nonlocal problem. As we know, for the autonomous system, Esteban et al. [8] first studied the existence and multiplicity of stationary solutions by using variational methods. After that, for the non-autonomous system of form

$$\begin{cases} -i\sum_{k=1}^{3}\alpha_k\partial_k u + (V(x)+a)\beta u + \omega u - \phi u = F_u(x,u), \\ -\Delta\phi = 4\pi|u|^2. \end{cases} \quad \text{in } \mathbb{R}^3 \tag{1.3}$$

Chen and Zheng [9] considered the non-periodic superlinear case with pure power nonlinearities, and the existence of least energy stationary solutions was obtained. An asymptotically linear non-periodic problem was considered in [7]. Recently, Zhang et al. [10,11] studied the existence of ground state solutions for periodic asymptotically linear case.

Motivated by the above facts, in this paper we are concerned with system (1.3) with non-autonomous superlinear nonlinearity and periodicity condition. To the best of our knowledge, it seems that there is almost no work on the existence and multiplicity of solutions for this case up to now. The main aim of this paper is to study the existence and multiplicity of solutions via variational methods. Before stating our main result, we first make the following assumptions