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Stability of a budworm growth model with random perturbations

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Abstract

A budworm growth model perturbed by both white noises and regime switchings is proposed and analyzed. It is proven that there is a threshold. If this threshold is positive, then the model has a unique ergodic stationary distribution; if this threshold is negative, then the zero solution of the model is stable. The results show that both white noises and regime switchings can change the stability of the model greatly. Several numerical simulations based on realistic data are also introduced to illustrate the main results.

Keywords: Budworm growth model, white noise, regime switching, stability, stationary distribution

1. Introduction

Budworm is spreading throughout much of the world. During its outbreak, millions of hectares of plants can be severely damaged, which can result in significant losses of food and timber resources, and the effects can last for decades ([1]). For example, the recent outbreak of spruce budworm in Quebec province of Canada had lasted for more than 10 years, and had spread through more than 7.1 million hectares; it may soon spread through eastern Canada and US ([2]). A classical budworm growth model is ([3, 4]):

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x\big[r - a_1 x\big] - \frac{a_2 x^2}{\theta + x^2}, \tag{1}$$

where $x = x(t)$ the size of the budworm population at time t; $r > 0$ is the growth rate; $a_1 > 0$ is the intra-specific competition rate; $a_2 > 0$ is the maximum consumption rate given by the predators or parasites of x; $\theta > 0$ measures the saturate effect of the predators or parasites at the high density of x. [3, 5, 6] studied the stability and oscillation of model (1). Liu and his coworkers [7, 8] investigated the persistence and extinction of model (1) with impulse. Bifurcation of model (1) with reaction diffusion was explored in [9]. Berezansky and Braverman [10] analyzed the stability of model (1) with time delay.

Since the natural world is full of random perturbations, then it is interesting to study the budworm growth model in random environments, and to reveal the effects of random perturbations on the dynamics of the model. To begin with, let us consider one type of environmental noise, i.e., telegraph noise. The telegraph noise is a switching between several regimes of environment [11]. It has been noted that the growth rate of budworm is often subject to regime switching. An example is that the growth rates of budworm in summer and autumn are quite different ([2]). Similarly, other parameters in model (1) are also often subject to regime switching. Several authors ([12]-[16]) have pointed out that the regime switching is memoryless, and can be modeled by a right-continuous Markov chain with finite states. Following this approach, model (1) becomes:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x\Big[r(\xi) - a_1(\xi)x(t)\Big] - \frac{a_2(\xi)x^2}{\theta(\xi) + x^2}, \tag{2}$$

where $\xi = \xi(t)$ is a right-continuous Markov chain with state space $\mathbb{S} = \{1, ..., m\}$.

Now let us consider another type of environmental noise, i.e., white noise, which is common in the natural world. There are several approaches to introduce white noise into population models. In this paper, following the approach in [12]-[16], we consider the parameter perturbation, i.e., we suppose that the parameters r and a_i $(i = 1, 2)$ are perturbed by white noises, with

$$r(\cdot) \to r(\cdot) + \sigma_1(\cdot)\dot{W}_1(t), \quad -a_i(\cdot) \to -a_i(\cdot) + \sigma_{i+1}(\cdot)\dot{W}_{i+1}(t),\ i = 1, 2,$$

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