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New existence results on periodic solutions of non-autonomous second order Hamiltonian systems

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Abstract

In this paper, we are concerned with the existence of periodic solutions for the following non-autonomous second order Hamiltonian systems

$$\begin{cases} \ddot{u}(t) + \nabla F(t, u(t)) = 0, \text{ a.e. } t \in [0, T], \\ u(0) - u(T) = \dot{u}(0) - \dot{u}(T) = 0, \end{cases}$$

where $F : \mathbb{R} \times \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ is T-periodic ($T > 0$) in its first variable for all $x \in \mathbb{R}^{\mathbb{N}}$. When potential function $F(t, x)$ is either locally in t asymptotically quadratic or locally in t superquadratic, we show that the above mentioned problem possesses at least one T-periodic solutions via the minimax methods in critical point theory, specially, a new saddle point theorem which is introduced in [M. Schechter, New linking theorems, Rend. Sem. Mat. Univ. Padova 99 (1998), 255-269].

Keywords: Periodic solutions; Locally in t asymptotically quadratic; Locally in t superquadratic; New saddle point theorem

1. Introduction and main results

Consider the second order non-autonomous Hamiltonian systems

$$\begin{cases} \ddot{u}(t) + \nabla F(t, u(t)) = 0, \quad \text{a.e. } t \in [0, T], \\ u(0) - u(T) = \dot{u}(0) - \dot{u}(T) = 0, \end{cases} \tag{1.1}$$

where $F : \mathbb{R} \times \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ is T-periodic ($T > 0$) in its first variable for all $x \in \mathbb{R}^{\mathbb{N}}$, $\nabla F(t, x) = \frac{\partial}{\partial x} F(t, x)$, and satisfies the following assumption:

(H_0) $F(t, x)$ is measurable in t for every $x \in \mathbb{R}^{\mathbb{N}}$ and continuously differentiable in x for a.e. $t \in [0, T]$, and there exist $a \in C(\mathbb{R}^+, \mathbb{R}^+), b \in L^1(0, T; \mathbb{R}^+)$ such that

$$|F(t, x)| \le a(|x|)b(t), \qquad |\nabla F(t, x)| \le a(|x|)b(t)$$

for all $x \in \mathbb{R}^{\mathbb{N}}$ and a.e. $t \in [0, T]$.

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