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Ultimate boundedness of impulsive fractional delay differential equations*

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Abstract

This paper concerns with the ultimate boundedness problem for impulsive fractional delay differential equations. Based on the impulsive fractional differential inequality, the boundedness of Mittag-Leffler functions, and the successful construction of suitable Lyapunov functionals, some algebraic criteria are derived for testing the global ultimate boundedness of the equations, and the estimations of the global attractive sets are provided as well. One example is also given to show the effectiveness of the obtained theoretical results.

Keywords: Impulsive fractional differential equations; Time delay; Ultimate boundedness; Mittag-leffler function; Lyapunov functional

1 Introduction

Impulsive fractional differential equations are an special class of hybrid dynamical systems which combine the characteristics of the impulsive systems and the fractional systems. With the development of the theory of the impulsive differential equations and the fractional differential equations, more and more scholars begin to give their attentions to the theory of impulsive fractional differential equations. Many interesting contributions have been made in the theory of impulsive fractional differential equations [1-12]. These results cover stability, boundedness, existence and uniqueness of impulsive fractional differential equations.

On the other hand, time delays widely exist in many practical systems, which may cause oscillation, divergence, chaos, instability or other undesirable performances in the systems. Therefore, it is necessary to analyze delay effects on the dynamical behaviors of impulsive fractional differential equations. Recently, some significant results on the impulsive fractional delay differential equations have been reported [13-21]. These reports only focused on the study of stability, existence and uniqueness, but besides stability, existence and uniqueness problems, another most important asymptotic property of dynamical systems is boundedness, which plays a major role in the study of the basic properties of the solutions such as existence, stability, persistence and attractivity. Therefore the boundedness should be discussed in the studies of impulsive fractional delay differential equations.

Unfortunately, to our best knowledge, no work has been reported on the boundedness of impulsive fractional delay differential equations in the literature. The motivation of this paper is to fill this gap. Based on the impulsive fractional differential inequality, the boundedness of Mittag-Leffler functions, and the successful construction of suitable Lyapunov functionals, some algebraic criteria are derived for testing the global ultimate boundedness of the equations.

2 Preliminaries

The symbol $|\cdot|$ stands for the Euclidean norm in $\mathbb{R}^n$, $\mathbb{R}_+ = [0, \infty)$, and $\mathbb{N} = \{1, 2, 3, ...\}$. The superscript '$T$' stands for the transpose for vectors or matrices. The symbols $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ stand for the maximum and the minimum eigenvalue of a symmetrical real matrix Q, respectively. In the following, we recall some important definitions of fractional calculus in [21].

Gamma function $\Gamma(z)$:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \; Re(z) > 0,$$

where the symbol $Re(z)$ denotes the real part of the complex number z.

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