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Weak Galerkin finite element method for a class of quasilinear elliptic problems☆

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Abstract

The purpose of this paper is to study the weak Galerkin finite element method for a class of quasilinear elliptic problems. The weak Galerkin finite element scheme is proved to have a unique solution with the assumption that guarantees the corresponding operator to be strongly monotone and Lipschitz-continuous. An optimal error estimate in a mesh-dependent energy norm is established. Some numerical results are presented to confirm the theoretical analysis.

Keywords: weak Galerkin finite element method, monotone quasilinear elliptic problems
2010 MSC: 65N30, 65N15, 35J50

1. Introduction

The weak Galerkin (WG) finite element method (FEM) [1, 2, 3] is a recently developed numerical method for solving partial differential equations. The main advantage of this method is that the discontinuous piecewise polynomials are allowed in the approximation solutions due to the introduction of discrete weak gradient. This make the WGFEM method more attractive and flexible in solving wide scope of partial differential equations, see e.g. [4, 5, 6, 7, 8, 9, 10].

In this paper, we shall give an analysis of the weak Galerkin finite element method for solving the following elliptic problem

$$\begin{aligned} -\nabla\cdot(a(x,|\nabla u|)\nabla u) &= f \quad \text{in } \Omega,\\ u &= 0 \quad \text{on } \partial\Omega, \end{aligned} \tag{1}$$

where Ω is a bounded open set with Lipschitz continuous boundary $\partial\Omega$ in $\mathbb{R}^2$, the function a satisfies the following assumptions.

(A) $a \in C(\overline{\Omega}\times[0,\infty))$ and there exist positive constants m_a and M_a, such that

$$m_a(t-s) \le a(x,t)t - a(x,s)s \le M_a(t-s), \quad t \ge s \ge 0, \quad x \in \overline{\Omega}. \tag{2}$$

(B) The function $a(x,t)$ is a continuously differentiable function with respect to x and the derivative is bounded.

We note that many numerical methods have been proposed and analyzed for solving this type of monotone quasilinear elliptic problem. The interested readers are referred to [11, 12, 13] for finite element methods, and [14, 15] for finite volume methods and discontinuous Galerkin finite element methods.

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