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# A note on crack propagation paths inside elastic bodies

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ABSTRACT

The note is concerned with a model of linear elastostatics for a two-dimensional inhomogeneous anisotropic body weakened by a single straight crack. On the crack faces, nonpenetration conditions/Signorini conditions are imposed. Relying upon a higher regularity result in Besov spaces for the displacement field in a neighborhood of the crack tip, we prove that the energy release rate is actually independent of the choice of a subsequent crack path (among the possible continuations of class $H^3$).
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## 1. Formulation of the problem

One of the frequently used approaches for describing quasistatic crack propagation in elastic bodies is the Griffith energy criterion [1]. If the possible crack path is known a priori, the Griffith energy criterion can be formulated in terms of the energy release rate, which is the negative of the first right derivative of the potential deformation energy with respect to the crack length, and the fracture toughness. This note is inspired by the article of M. Brokate and A. Khludnev [2] in which they investigated the dependence of the energy release rate on smooth enough crack propagation paths for two-dimensional homogeneous isotropic linear elastic bodies with traction-free crack faces (pure Neumann conditions) and extends their results to the case of inhomogeneous anisotropic elastic bodies with cracks subjected to nonpenetration conditions. More precisely, for the case of a single straight crack, we prove that the energy release rate does not depend on a subsequent crack path, provided that the crack propagates along a smooth curve having the same initial tangential vector and curvature as at the crack tip. The key observation which allows us to work with curved cracks of class $H^3$ instead of $H^4$, see [3], is that in a neighborhood of the crack tip, the displacement field is $B^{3/2}_{2,\infty}$ regular [4]. This higher regularity result was applied to the study of computational aspects of quasistatic crack propagation in [5]. Finally, we mention the paper [6] in which, for the setting of linear antiplane elastostatics, it was shown that the energy release rate is continuous with respect to the Hausdorff

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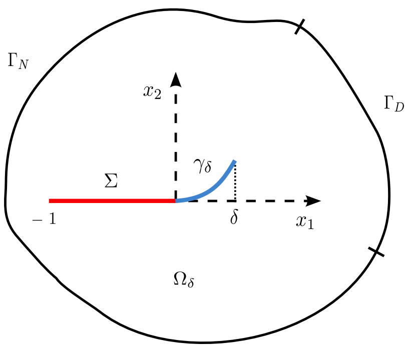


**Fig. 1.** The geometrical setup of the problem.

convergence in a class of admissible crack paths and is actually independent of the choice of the curve that extends the crack (among the possible continuations of class $C^{1,1}$) .

We next turn to a precise formulation of our model. Let $\Omega$ be a bounded domain in $\mathbb{R}^2$ with Lipschitz boundary corresponding to the reference configuration of an undamaged physical body. The boundary $\partial\Omega$ is a union of two disjoint subsets $\Gamma_D$ and $\Gamma_N$, with $\mathcal{H}^1(\Gamma_D) > 0$, where $\mathcal{H}^1$ denotes the one-dimensional Hausdorff measure. On the Dirichlet part of the boundary $\Gamma_D$ the displacement field is prescribed, and on the Neumann part $\Gamma_N$ the traction is imposed. The set $\Sigma = [-1, 0] \times \{0\}$ is an initial crack, while a subsequent crack path is represented by the graph of a smooth function $\psi$, i.e. $\gamma_\delta = \{ (x_1, \psi(x_1)) \mid x_1 \in [0, \delta] \}$, $\delta \in [0, \delta_*]$, with $\psi(0) = \psi_{,1}(0) = \psi_{,11}(0) = 0$. A subscript following a comma indicates differentiation with respect to that Cartesian coordinate. The whole crack $\gamma^\delta = \Sigma \cup \gamma_\delta$ is assumed to be such that $\gamma_\delta \subset \Omega$ for all $\delta \in [0, \delta_*]$, and the set $\Omega_\delta = \Omega \setminus \gamma^\delta$ is the domain with crack $\gamma^\delta$, see Fig. 1. Moreover, we assume that the domain $\Omega$ is split into two subdomains $\Omega_1$ and $\Omega_2$ with Lipschitz boundaries such that $\gamma_{\delta_*} \subset \partial\Omega_1 \cap \partial\Omega_2$ and $\mathcal{H}^1(\partial\Omega_i \cap \Gamma_D) > 0$, $i = 1, 2$. This guarantees that the first Korn inequality is valid in the non-Lipschitz domains $\Omega_\delta$ for all $\delta \in [0, \delta_*]$.

To formulate the model in a variational setting, we introduce the convex closed cone of kinematically admissible displacements $K(\Omega_\delta) = \{ v \in H^1(\Omega_\delta)^2 \mid [v_i]\nu_i \geq 0 \text{ on } \gamma^\delta,\ v = 0 \text{ on } \Gamma_D \}$, where $[v] = v^+ - v^-$ is the jump of a function $v$ across the crack $\gamma^\delta$, with the signs $\pm$ corresponding to the positive and negative directions of the unit normal vector $\nu$ on $\gamma^\delta$. A repeated subscript is to be summed over the values 1, 2. Neglecting body forces, the potential deformation energy of the system is given by the functional

$$\mathcal{E}_\psi(v;\delta) = \frac{1}{2}\int_{\Omega_\delta} a_{ijkl}\varepsilon_{kl}(v)\varepsilon_{ij}(v)\,\mathrm{d}x - \int_{\Gamma_N} f_i v_i\,\mathrm{d}\mathcal{H}^1.$$

Here, $f = (f_1, f_2) \in L^2(\Gamma_N)^2$ is the traction acting on $\Gamma_N$ and $\varepsilon = \{\varepsilon_{ij}(v)\}$ is the infinitesimal strain tensor, $2\varepsilon_{ij}(v) = v_{i,j} + v_{j,i}$, $i, j = 1, 2$. The fourth order tensor $A = \{a_{ijkl}\}$ is the elastic modulus tensor satisfying the usual properties of symmetry and positive definiteness and the regularity condition $a_{ijkl} \in C^1(\overline{\Omega})$, $i, j, k, l = 1, 2$. Then the minimization problem for determining the displacement field $u^\delta$ corresponding to the domain $\Omega_\delta$ reads as follows:

$$\text{Find} \quad u^\delta \in K(\Omega_\delta) \quad \text{such that} \quad \mathcal{E}_\psi(u^\delta;\delta) \leq \mathcal{E}_\psi(v;\delta) \quad \text{for all} \quad v \in K(\Omega_\delta). \tag{1}$$

The coercivity, weakly lower semicontinuity, and strong convexity of $\mathcal{E}_\psi$ ensure that there exists a unique solution of (1) satisfying the variational inequality

$$u^\delta \in K(\Omega_\delta), \quad \int_{\Omega_\delta} a_{ijkl}\varepsilon_{kl}(u^\delta)\varepsilon_{ij}(v - u^\delta)\,\mathrm{d}x \geq \int_{\Gamma_N} f_i(v_i - u_i^\delta)\,\mathrm{d}\mathcal{H}^1 \quad \text{for all} \quad v \in K(\Omega_\delta). \tag{2}$$

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