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Bayesian inference with overlapping data: Reliability estimation of multi-state on-demand continuous life metric systems with uncertain evidence

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A R T I C L E I N F O

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A B S T R A C T

A Bayesian system reliability estimation methodology for multiple overlapping uncertain data sets within complex multi-state on-demand and continuous life metric systems is presented in this paper. Data sets are overlapping if they are drawn from the same process at the same time, with reliability data from sensors attached to a system at different functional and physical levels being a prime example. Treating overlapping data as non-overlapping loses or incorrectly infers information on system reliability. Methodologies for system reliability analysis of certain overlapping data sets have previously been proposed. These methodologies, and the approach presented in this paper, are able to incorporate overlapping uncertain evidence from systems with a detailed understanding of the system logic represented using fault-trees, reliability block diagrams or equivalent representations. The method presented here builds on approaches that have already been developed by the authors that allow incorporation of exact or certain data sets.

Introduction

Many system reliability analysis methodologies focus on system failure logic (such as that represented in *reliability block diagrams* or *fault-trees*) to express system failure probability in terms of subordinate component failure probabilities. Such methodologies are explained in virtually all texts on systems reliability: for example Hoyland and Rausand [1], Hiromitsu and Henley [2] and the Nuclear Regulatory Committee (NRC) Probability Risk Assessment Guide [3]. These techniques promote system reliability as a function of component reliabilities, which in turn direct focus to reliability testing and data collection at component level. Component data are then used to develop component level reliability estimates, with which system level reliability values are calculated. This approach automatically precludes useful system and sub-system data, which is referred to here as *higher level data* as it appears ‘higher’ in many visualization methodologies such as fault trees.

Generally, systems can be of two types: ‘demand-based’ or ‘continuously operating’ (noting that mixed types are also possible). Demand-based or on-demand systems are subjected to discrete demands or trials and respond by operating (or existing) within certain discrete states. The simplest of on-demand systems are ‘binary-state’ systems, where components are either in the ‘functional’ or ‘failed’ states. ‘Multi-state’ systems involve components that can be classified by order of severity in various degraded states ranging from ‘functional’ to ‘failed.’ Methods for reliability analysis of such systems can be found in the literature (see for example [4] and [5]). Systems based on continuous life metrics are those whose failure probability is an explicit function of an independent life variable such as time or distance.

Jackson and Mosleh [6–9] developed Bayesian methodologies for incorporating higher level data in on-demand systems and continuous life metric systems. A system often involves multiple sensors (sensors are broadly defined as monitoring points through data gathering devices; human, machine, or otherwise). This means that reliability data sets are often overlapping in nature. Sets of overlapping data meet the following criteria:

1. **Simultaneity** – the data sets are drawn from observations or demands that occur at the same time; and
2. **Correspondence** – the data sets result from the same system or process.

Initially, only approximate Bayesian methodologies were developed for Bayesian analysis of higher-level data [10–13]. These methods have been generalized further, particularly for continuous multi-state systems in [14,15] that place bounds on

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Nomenclature

a	reference number of a particular combination of an on-demand system's state vectors
α_b^x	xth parameter of the Dirichlet distribution used as a conjugate prior distribution of state probabilities (for on-demand systems)
$\tilde{\boldsymbol{\alpha}}_b$	bth compont's vector of it Dirchlet conjugate prior distribution of state probabilities
b	reference number of a particular component within a system
E	evidence set
$\hat{E}$	observed evidence set
(t)	unit or Heaviside step function
i	reference number of a particular sensor within a system
j	reference number of a particular component type within a system
$k_i^{S(x)}$	number of times the ith sensor detects the xth functional state (for on-demand systems)
$\tilde{k}_i^S$	vector of possible numbers of times the ith sensor detects the xth functional state (for on-demand systems)
$\boldsymbol{l}$	reference number of a particular state vector of an on-demand system
$L(E\|\boldsymbol{\theta})$	likelihood function – the likelihood of observing evidence set E given parameter set $\boldsymbol{\theta}$
m	number of sensors within a system
n	number of component types within a system
n'	number of components within a system
$p_b^{(x)}$	probability of the bth component being in the xth (for on-demand systems)
$p_i^{S(x)}$	probability of the ith sensor detecting the xth functional state (for on-demand systems) which is expressed as function that includes subordinate component state probabilities as inputs
$\tilde{\boldsymbol{p}}_b$	bth component's vector of all state probabilities – $p_b^{(x)}$ (for on-demand systems)
$\boldsymbol{p}$	set of all component's state probabilities for an on-demand system
$\pi_0(\boldsymbol{\theta})$	prior distribution of the parameter set $\boldsymbol{\theta}$
$\pi_1(\boldsymbol{\theta}\|E)$	posterior distribution of the parameter set $\boldsymbol{\theta}$ given evidence set E
r	number of demands an on-demand system is subjected to
r_i^S	number of demands the ith sensor observes (for on-demand systems)
$\phi_i^S(\tilde{\boldsymbol{x}})$	structure function of the ith sensor for a particular component state vector, $\tilde{\boldsymbol{x}}$
t_i^S	time at which the ith sensor detects failure (for time-based continuous systems)
$\tilde{\boldsymbol{t}}^S$	vector of all sensor failure detection times (for time-based continuous systems)
$\hat{t}_i^S$	estimated time at which the ith sensor detects failure (for time-based continuous systems)
$\approx t^s$	vector of all estimated sensor failure detection times (for time-based continuous systems)
Δt_i^S	uncertainty interval of the time at which the ith sensor detects failure (for time-based continuous systems)
$\Delta\tilde{\boldsymbol{t}}^S$	vector of all sensor uncertainty intervals (for time-based continuous systems)
$\tilde{\boldsymbol{t}}_i^{\subset S}$	set of failure detection times of sensors inferentially subordinate to the ith sensor (for time-based continuous systems)
$\tilde{\theta}_b$	vector of bth component's reliability parameters
θ	set of all components' reliability parameters
$\boldsymbol{\theta}_i^{\subset S}$	set of parameter sets of component inferentially subordinate to the ith sensor (for time-based continuous systems)
v_l	number of occurrences of the lth state vector for an on-demand system subject to r demands
$\tilde{\boldsymbol{v}}$	vector containing a combination of r state vectors (for on-demand systems)
$(v_l)_a$	number of occurrences of the lth state vector in the ath combination of r state vectors (for on-demand systems)
$\tilde{\mathbf{v}}_a$	vector containing the ath combination of r state vectors (for on-demand systems)
$\mathbf{v}_E$	set of state vector combinations ($\tilde{\boldsymbol{v}}$) that imply the evidence set, E (for on-demand systems)
x	state of a particular component (for on-demand systems)
x_b	state of the bth component (for on-demand systems)
$\tilde{\boldsymbol{x}}$	state vector of a system containing all component states of an on-demand system
$(x_b)_l$	state of the bth component of the lth state vector of an on-demand system
$\tilde{\boldsymbol{x}}_l$	lth state vector of an on-demand system
z	number of allowable states for a system (for on-demand systems)

parametermoments, noting that the underlyhing mathematics remains particularly onerous. Multiple methodologies that can incorporate higher-level non-overlapping data have since been developed and are discussed in detail below [16–18]. Jackson and Mosleh discuss the error that is introduced when incorrectly analyzing overlapping data by constraining it to be considered as non-overlapping [6–9]. Graves et al. [19] proposed a method that incorporates overlapping data for multi-state on-demand systems. The methodology considers each demand in isolation (i.e. sensor states for each demand must be known), but cannot incorporate data that summarizes multiple demands on the system. The methodology proposed by Jackson and Mosleh [9] is completely generalized and able to consider multiple demands for multi-state systems.

This paper develops fully Bayesian methodologies for incorporating uncertain overlapping higher level data using techniques discussed above. In the case of on-demand systems, uncertain data manifests itself in terms of the uncertainty in number of observed failures from demands. For continuously operating systems, it is manifested in terms of the uncertainty in the time at which failure is detected. The latter scenario for continuously operating systems, a likelihood function that is not only computationally simpler than that proposed by Jackson and Mosleh in [7], but correctly replicates reality as all detection times have uncertainty expressed as the accuracy of the timing devices. This allows utility in applications where system reliability is periodically checked over specific time intervals (such as the case of maintenance schedules) or at specific points of operations (such as the case of sequential