Accepted Manuscript

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PII: S0893-9659(17)30365-8
DOI: https://doi.org/10.1016/j.aml.2017.12.004
Reference: AML 5390

To appear in: *Applied Mathematics Letters*

Received date : 8 September 2017
Revised date : 5 December 2017
Accepted date : 5 December 2017

Improving the pressure accuracy in a projection scheme for incompressible fluids with variable viscosity.☆

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Abstract

The *incremental projection scheme* and its enhanced version, the *rotational projection scheme* are powerful and commonly used approaches producing efficient numerical algorithms for solving the Navier-Stokes equations. However, the much improved rotational projection scheme cannot be used on models with non-homogeneous viscosity, imposing the use of the less accurate incremental projection. This paper present a projection method for the Navier-Stokes equations for fluids having variable viscosity, giving a consistent pressure and increased accuracy in pressure when compared to the incremental projection. The accuracy of the method will be illustrated using a manufactured solution.

Keywords:
Navier-Stokes equations, heterogeneous viscosity, projection methods, fractional step methods, finite element.
2000 MSC: 76D05, 65M60, 35Q35

1. Introduction

Solving the incompressible Navier-Stokes equations for fluid of homogeneous nature is, in itself, a difficult task. Even "simple fluid", in three dimensional domain are challenging with today's numerical tools and technology.

One way to circumvent the intrinsic saddle point structure due to the incompressibility constraint of the Navier-Stokes equation, is to follow the pioneering works of Chorin [1] and Temam [2] who introduced projection methods. The idea, based on the Helmoltz decomposition (see [3] for instance), is to introduce a fractional time step to decouple the incompressibility constraint from the diffusion operator. Numerous variants have been proposed over the years, for an interesting overview, we refer to Guermond et al. [4].

The first major improvement of the original projection scheme was proposed by Goda [5]. Although frequently used, this *incremental projection scheme* induces an artificial (inconsistent) boundary condition on the resulting pressure. The second important refinement of the projection method is the *rotational projection scheme* proposed by Timmermans et al. [6]. This gives a consistent boundary condition for the pressure and improves the accuracy of the approximation of the pressure (see [7]). Once again we refer the reader to [4] for a detailed review of the various form of the projection method and its error estimates.

Fluid mixture models based on Allen-Cahn or Cahn-Hilliard and certain cases of natural convection are examples of incompressible fluids where the viscosity depends on an external quantities (phase field, temperature, etc.) which are, a priori, time depend and nonhomogeneous.

Unfortunately, in case of a non-homogeneous viscosity, the rotational projection scheme leads to erroneous pressure (see Figure 1). Therefore the incremental projection (and its artificial boundary condition on the

☆This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).
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