Existence of the solutions of a reaction cross-diffusion model for two species

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Abstract

In this paper, we investigate a two-cooperation species model with reaction cross-diffusion under the Dirichlet boundary value condition. By applying Lyapunov-Schmidt reduction method and some important formulas, we obtain the existence of coexistence solutions under some given conditions.

Keywords: Cooperative species; cross-diffusion; semi-trivial steady state; coexistence solution; Lyapunov-Schmidt reduction.

1. Introduction

Recently, the dynamics models in the form of reaction cross-diffusion systems have received considerable researchers' attentions, one can refer to [1, 2, 8, 9]. Ecologically, positive solutions correspond to the existence of steady states of species. So the set of positive solutions may contain crucial clues for the stationary patterns. Kuto and Yamada[1] studied the following prey-predator model with cross-diffusion:

$$\begin{cases} -\Delta[(D_1 + \rho_{12} v)u] = u(\lambda_1 - a_1 u - b_1 v), & \text{in } \Omega, \\ -\Delta[(D_2 + \rho_{21} u)v] = v(\mu_1 - d_1 u + c_1 v), & \text{in } \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain with a smooth boundary, $u(x)$ and $v(x)$ denote the population density of two species at location x, which are interacting and migrating in the same habitat Ω, $a_1, b_1, c_1, d_1 > 0, D_j > 0 (j = 1, 2)$ and $\lambda, \mu \in \mathbb{R}$. Here, D_j represents the diffusive coefficient and ρ_{12}, ρ_{21} are considered as cross-diffusion pressures. Here, λ_1 and μ_1 denote the growth rates of the species, respectively, a_1 and d_1 are the mortal rates of u and v, respectively, b_1 and d_1 are the interaction rates between the two species. By some rescaling the forms as follows:

$$(\widetilde{u}, \widetilde{v}) = (\frac{a_1}{D_1} u, \frac{c_1}{D_2} v),$$

$$(\alpha, \beta, \lambda, \mu, b, d) = (\frac{\rho_{12} D_2}{c_1 D_1}, \frac{\rho_{21} D_1}{a_1 D_2}, \frac{\lambda_1 \mu_1}{D_1 D_1 2}, \frac{b_1 D_2}{d_1 D_1}, \frac{c_1 D_1}{a_1 D_2}),$$

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