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On symmetric block triangular splitting iteration method for a class of complex symmetric system of linear equations

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Abstract

For solving a class of complex symmetric linear system, we first transform the system into a block two-by-two real formulation and construct a symmetric block triangular splitting (SBTS) iteration method based on two splittings. Then, eigenvalues of iterative matrix are calculated, convergence conditions with relaxation parameter are derived, and two optimal parameters are obtained. Besides, we present the optimal convergence factor and test two numerical examples to confirm theoretical results and to verify the high performances of SBTS iteration method compared with two classical methods.

Mathematics Subject Classifications: 65F10; 65F50

Keywords: Complex linear system; Symmetric positive definite; Block triangular; Splitting; Convergence

1. Introduction

Consider the complex system of linear equations

$$Au = b,\ A \in \mathbb{C}^{n\times n},\ u, b \in \mathbb{C}^n \tag{1.1}$$

where the symmetric matrix $A = W + \imath T$ and $\imath = \sqrt{-1}$. If let $u = x + \imath y$ and $b = p + \imath q$ with $x, y, p, q \in \mathbb{R}^n$, then the above linear system can be rewritten as

$$\mathcal{A}z = \begin{pmatrix} W & -T \\ T & W \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} =: \widetilde{b}, \tag{1.2}$$

where $W, T \in \mathbb{R}^{n\times n}$ are both symmetric matrices with at least one of them being positive definite. Hereafter, without loss of generality, we assume that W is positive definite. As we all know, the equations (1.1) arises frequently in many scientific computations and engineering applications, one can refer to [1–4].

Recently, lots of effective iterative methods have been proposed in the literatures for solving the linear system (1.1) or its real equivalent formulation (1.2), such as preconditioned Krylov subspace iteration methods [3, 5], conjugate gradient iteration method [6], MHSS iteration method [7] and its variants [8, 9]. Moreover, the C-to-R iteration methods [10–12], GSOR and shift-splitting iteration method [13–15] are also useful techniques for solving complex symmetric problem. As a matter of fact, the linear equations (1.2) can be regarded as a special case of generalized saddle point problems [16]. Furthermore, we have known that recent studies proposed an upper and lower triangular(ULT) iteration and its parameterized variant (PULT) for nonsingular saddle point problems ([17, 18]), the authors analyzed the characteristic of the eigenvalues as well as the convergence conditions, so were numerical results there to show the effectiveness.

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