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Recovering a source term in the time-fractional Burgers equation by an energy boundary functional equation

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Abstract

An inverse source problem for the recovery of an unknown space-time dependent source term of a time-fractional Burgers equation is solved in the paper. By using the prescribed boundary data, a sequence of boundary functions are derived, which together with the zero element constitute a linear space. An energy boundary functional equation is derived in the linear space, of which the time-dependent energy is preserved for each energy boundary function. The iterative algorithm used to recover the unknown source with energy boundary functions as the bases is developed, which is robust and convergent fast.

Keywords: Time-fractional Burgers equation, Space and time dependent source, Energy boundary functions, Energy functional equation, Iterative method

1. Introduction

The time-fractional Burgers equation is a typical nonlinear anomalous subdiffusion convection equation [1], which has a broad applications. Burgers' equation has been of considerable physical interest because it is an appropriate simplification of the Navier-Stokes equations, and is also the governing equation of a number of one-dimensional flow systems, including the convection and diffusion of heat, weak shock propagation, and compressible turbulence. The applications and computations of the time-fractional Burgers equation can be found in [2, 3, 4].

We consider

$$D_t^\alpha u(x,t) - u(x,t)u_x(x,t) = u_{xx}(x,t) + H(x,t), \quad 0 < x < \ell, \quad 0 < t \le t_f, \tag{1}$$

$$u(0,t) = F_0(t), \quad u(\ell,t) = F_\ell(t), \tag{2}$$

where $H(x,t)$ is an unknown source term to be recovered, and $F_0(t)$ and $F_\ell(t)$ are, respectively, the given left-boundary condition and right-boundary condition. The time-fractional