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Integral condition for oscillation of half-linear differential equations with damping

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Abstract

The purpose of this paper is to provide an oscillation theorem that can be applied to half-linear differential equations with time-varying coefficients. A parametric curve by the coefficients is focused in order to obtain our theorem. This parametric curve is a generalization of the curve given by the characteristic equation of the second-order linear differential equation with constant coefficients. The obtained theorem is proved by transforming the half-linear differential equation to a standard polar coordinates system and using phase plane analysis carefully.

Key words: Oscillation; Half-linear differential equation; Phase plane analysis
2010 MSC:

1. Introduction

This paper is concerned with an oscillation theorem for the second-order nonlinear differential equation with a damping term,

$$\left(\Phi_p(x')\right)' + a(t)\Phi_p(x') + b(t)\Phi_p(x) = 0, \tag{1}$$

where a and b are locally integrable functions on $[0,\infty)$ and Φ_p is a real-valued function defined by

$$\Phi_p(z) = \begin{cases} |z|^{p-2}z & \text{if } z \neq 0, \\ 0 & \text{if } z = 0. \end{cases}$$

with a real number $p > 1$. Equation (1) has the trivial solution $(x, x') \equiv (0, 0)$. When $p = 2$, equation (1) becomes the linear homogeneous differential equation with variable coefficients,

$$x'' + a(t)x' + b(t)x = 0. \tag{2}$$

It is well-known that all solutions of (1) are unique for given initial conditions and continuable in the future as well as those of (2) are (see, for example, [2, 5]). In addition to this property, many commonalities are seen in the asymptotic behavior of solutions of (1) and (2), such as oscillation and stability. For example, see [1, 3, 7, 8, 10, 13, 14, 15, 16, 19, 20, 21]. Equation (1) is one of half-linear differential equations. About half-linear differential equations, refer to the monograph [4] and the references therein.

Since all solutions of (1) are continuable in the future, they can be classified into two groups as follows: a nontrivial solution x of (1) is said to be *oscillatory* if there exists a sequence $\{t_n\}$ tending to ∞ such that $x(t_n) = 0$; otherwise, it is said to be *nonoscillatory*.

Let $u = a(t)$ and $v = b(t)$. Then, the point $(a(t), b(t))$ is considered to move in the (u, v)-plane. Let us call that trajectory a *parametric curve*. We divide the first quadrant of the (u, v)-plane into two regions by the curve $v = (u/p)^p$:

$$R_1 = \{(u, v) \colon u \geq 0 \text{ and } 0 \leq v \leq (u/p)^p\};$$

$$R_2 = \{(u, v) \colon u \geq 0 \text{ and } v > (u/p)^p\}.$$

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