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On the Projection-based Commuting Solutions of the Yang-Baxter Matrix Equation☆

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Abstract

We study the commuting solutions of the Yang-Baxter matrix equation $AXA = XAX$ when A is an arbitrary square matrix. By characterizing its commuting solutions based on projection matrices, we show that projections can be determined by using the generalized eigenspaces corresponding to the eigenvalues of A. Therefore, commuting solutions can be constructed explicitly. Our results are more general than those obtained recently by Dong [Appl. Math. Lett. 64 (2017), 231-234], Ding and Zhang [Appl. Math. Lett. 35 (2014), 86-89], and Ding and Rhee [J. Math. Anal. Appl. 402 (2013), 567-573.].

Keywords: Projection, eigenvalues, Yang-Baxter equation, Jordan form.
AMS classification: 15A18, 15A24, 65F15.

1. Introduction

The main aim of this paper is to construct projection-based commuting spectral solutions of a nonlinear matrix equation

$$AXA = XAX, \tag{1.1}$$

where A is an arbitrary $n \times n$ complex matrix and X is the unknown matrix to be determined. This nonlinear equation has been referred to as the Yang-Baxter-like matrix equation because it has a similar format to the classical Yang-Baxter equation [1, 12]. In 1967, Yang [12] first considered and studied a one-dimensional quantum mechanical many body problem with a combination of delta functions as the potential, and found a factorization of the scattering matrix. The matrix equation was obtained as a consistence property for the factorization. In 1972, Baxter [1] solved a eight-vertex model in statistical mechanics and obtained the same matrix equation. Then the Yang-Baxter equation was stated, and it has been extensively investigated by mathematicians and physicists in knot theory, braid group theory, and quantum group theory in the last thirty years; see for instance [8, 9, 10, 13] and the references therein. Recently, the nonlinear matrix equation in (1.1) has also been studied, see for example [2, 3, 4, 5, 6, 7, 11, 14, 15, 16].

In general, it is difficult to characterize and determine all the solutions in (1.1) for general A. Indeed, we can reformulate (1.1) into a system of polynomial equations. More precisely, it is equivalent to solving a system of n^2 quadratic polynomial equations in n^2 variables. This is a mathematically and computationally challenging problem. There are several methods for constructing some solutions of (1.1) based on structure of A. For example, researchers studied A where A is diagonalizable [7], a rank-one matrix [11] or a rank two matrix [16]. One interesting class of solutions of (1.1) is commuting solutions with respect to A, i.e., $AX = XA$. Some recent papers have been devoted to finding various commuting solutions of (1.1) under different assumptions on A. In particular, corresponding to each eigenvalue of A, a spectral projection solution was obtained in [2]. For a semi-simple eigenvalue with its multiplicity more than one, a nonlinear

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