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Ground state solution for a fourth-order nonlinear elliptic problem with logarithmic nonlinearity *

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Abstract

In this paper, we study the existence of ground state solutions of nonlinear elliptic equation with logarithmic nonlinearity by the Linking theorem and logarithmic Sobolev inequality. Our results are quite different from those in the case of polynomial nonlinearity.

Keywords: Fourth-order elliptic equation, Logarithmic nonlinearity, Ground state solution

MSC2010: 35J35, 35J66

1 Introduction and main result

In this paper, we study the following fourth-order nonlinear equations with logarithmic nonlinearity

$$\begin{cases} \Delta^2 u + c\Delta u = u\log|u| & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Δ^2 denotes the biharmonic operator, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. Let λ_1 be the principle eigenvalue of $-\Delta$ in $H_0^1(\Omega)$ and assume the parameter $c < \lambda_1$.

In recent years, great attention has been paid on the study of the fourth-order semilinear elliptic problem. For instance, in [6] the authors considered the problem with the nonlinearity $b[(u+1)^+ - 1]$ and pointed out that such kind of nonlinearity furnishes a model to study traveling waves in a suspension bridge. After that, lots of papers are devoted to the problem with more general nonlinearity. In [15, 18] the authors studied the existence of nontrivial solutions and sign-changing solutions when f is superlinear or sublinear at infinity. When f is asymptotically linear at infinity, three nontrivial solutions were obtained in [10] by applying the Morse theory. See also [1] for the existence of single nontrivial solution. For other interesting results of biharmonic equations, we refer to [7, 8] and the reference therein.

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