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Warranty claims forecasting based on a general imperfect repair model considering usage rate

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ABSTRACT

Because manufacturers of products sold with any type of warranty incur additional costs from servicing customer warranty claims, accurately forecasting the number of warranty claims has become increasingly important to manufacturers. Under the assumption of Weibull distributed time to failure, we propose two models based on a generalised renewal process that consider product usage rate to forecast the number of failures of products sold with a two-dimensional warranty policy. The accelerated failure time model is used to investigate the effects of the product usage rate on system reliability. The maximum likelihood estimation combined with a nonlinear constrained programming method is used to estimate the parameters of the proposed models. We conduct data experiments based on both simulation and real data collected from an excavator manufacturer in China to test the performance of the proposed models. The results indicate that the models incorporating a variable usage rate more accurately forecast the number of failures than those models based on a nominal usage rate.

1. Introduction

The use conditions under which the sold products operate are important factors related to the reliability of the sold products, also known as field reliability. To understand and estimate the field reliability of sold products are important to both the engineers and managers of the manufacturers. Nowadays, durable products are often sold with warranty and a large amount of warranty data are collected in warranty service operations. The warranty data analysis, which is a set of methods used to attract useful information from warranty data, provides a way to model and evaluate the field reliability of sold products. A warranty is a contractual obligation that requires the manufacturer, vendor, or seller to rectify problems or failures that occur in the warranty coverage through either repair or replacement [1]. The purpose of a warranty is to assure customers that the product will perform its intended function during the warranty period under normal usage conditions [2]. Accordingly, any type of warranty will generate potential costs for a manufacturer. In general, warranty costs may vary from 2 to 10 per cent of the sales price, depending on the product and the manufacturer [3]. Thus, accurately forecasting the number of warranty claims enables manufacturers to prepare financial plans, formulate optimal warranty policies, and plan for the storage of spare parts.

In recent years, many scholars have taken a great deal of research on approaches to warranty claims forecasting for repairable products under different assumptions. Some studies have been undertaken based on the assumption that the failed products are perfectly repaired, i.e., the failed products are restored to a condition that is "as-good-as-new". Models based on such assumption were discussed in many papers such as Wu et al. [4], Chien [5], Rao [6], Chien and Zhang [7], Liu et al. [8]. In practice, failed products are often not restored to an “as-good-as-new” condition. A simplified assumption is that the failed products are minimally repaired, i.e., the products are restored to a “same-as-old” condition upon repair. Based on such assumption, methods have been studied by several authors, such as Wu and Akbarov [9–11], Huang et al. [12], Wang et al. [13]. However, in both theory and practice, these two conditions are often of limited use for practical purposes because most repair services result in neither of these two extremes but in the more complex intermediate condition of imperfect repair [14]. Imperfect repair is defined as repair services that transform a failed product to an intermediate condition between an “as-good-as-new” and an “as-bad-as-old” condition. In the literature on imperfect repair, Kijima and Sumita [15] and Kijima [16] proposed the Kijima I and Kijima II models using a generalised renewal process (GRP) that incorporates the concept of “virtual age”. Based on Kijima and Sumita [15], Kaminskiy and Krivtsov [17], and Krivtsov [18] proposed a numerical solution for the Kijima model I by using a Monte Carlo (MC) simulation technique. Yanez et al. [19] combined MC simulation with numerical methods for maximum likelihood estimation (MLE) and a Bayesian

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method of parameter estimation for the Kijima model I. Mettas and Zhao [20] proposed a method to resolve the problems associated with the Kijima model II for single and multiple repairable systems. Jacopino et al. [21] used a Monte Carlo Markov Chain with slice sampling to model imperfect inspection and maintenance for the Kijima model I. Veber et al. [22] proposed the Kijima model I based on a finite Weibull mixture and used the Expectation Maximisation algorithm to estimate the unknown parameters of the model. Wang and Yang [23] developed a numerical method for a Weibull GRP and used a nonlinear constrained programming approach to estimate the parameters of the Kijima model I and Kijima model II. Yevkin and Krivtsov [24–25] proposed an approximate solution for a generalised renewal equation and comparatively analysed optimization policies under the GRP with an underlying Weibull distribution. Tanwar et al. [26] conducted a survey for imperfect repair of repairable systems using GRP based on arithmetic reduction of age and arithmetic reduction of intensity concepts in general and Kijima models in particular.

The above studies focus on a situation in which products are sold with a one- dimensional warranty policy. However, products such as automobiles, printers, and excavators are frequently sold with a two-dimensional warranty policy, in which the warranty period is simultaneously determined by both age and usage. For the two-dimensional warrantied products, both age and usage rate are important factors to reliability of products and should be considered in warranty claims forecasting. To the best of our knowledge, little research has been conducted involving warranty claims forecasting considering the usage rate based on a GRP model. Inspired by the work of Kaminskiy and Krivtsov [17], we propose two models for predicting the warranty claims of products sold with a two-dimensional warranty policy considering the usage rate.

The key contribution of this study is that we propose two GRP-based models for warranty claims forecasting of products sold with two-dimensional warranty policy by taking both the age and the usage rate into consideration, which are extensions of the existing age based GRP models. Since the complexity of the proposed models, constrained maximum likelihood estimator with simulation based optimization method is used for parameters estimation. We also validate the proposed models using real field failure data collected from an excavator manufacturer of China.

The rest of this paper is organised as follows: Section 2 presents an overview of probabilistic models for repairable systems. Section 3 presents the proposed GRP- based models considering the usage rate and the MLE method for estimating the parameters in the proposed models. In Section 4, data experiments based on both simulation and real data are conducted to test the performance of the proposed models. Section 5 draws conclusions and gives some future topics worth investigating in this area.

2. Approaches to the reliability analysis on repairable systems

In this study, we assume that all products are repairable and sold with a two-dimensional warranty policy; we also assume that any failure produces an immediate claim and that the repair time is omitted such that the failure can be viewed as a point process. Furthermore, we assume that all warranty claims are valid, i.e., the warranty claims are all caused by real failures. We select a Weibull distribution as the underlying distribution because it is commonly used for the distribution of time to failure in reliability modelling and its flexibility and applicability to various failure processes [19].

We consider a repairable product observed beginning at start time $t_0 = 0$ to end time T. Let t be the successive failure times (i.e., actual age) denoted by $t_1, t_2, \ldots, t_n$, where n is the number of failures a product experienced. Let x be the time between failures and referred to as $x_1, x_2, \ldots, x_n$, we have $x_i = t_i - t_{i-1} (i = 1, 2, \ldots, n)$.

Three types of stochastic processes are generally used in reliability analyses of repairable products sold with two-dimensional warranties.

If a repairable product in service can be restored to an “as-good-as-new” condition, then the failure process is termed an ordinary renewal process (ORP), which is defined as a process in which the times between failures are regarded as independent and identically distributed. In general, application of the ORP model to the reliability analysis of repairable products is restricted unless the product consists of one non-repairable component in a socket (i.e., the failed component is always replaced with a new component when a product component fails) [19]. In contrast to ORP models, non-homogeneous Poisson process (NHPP) models assume that the product is repaired to an “as-bad-as-old” condition. Both ORP and NHPP models have been successfully used to study and evaluate repairable products. From the perspective of practical reliability engineering, a system is expected to return to an intermediate condition between the “as-good-as-new” condition and “as-bad-as-old” condition after a repair. Hence, the GRP model was proposed under an assumption that the product returns to an intermediate “better-than-old-but-worse-than-new” condition after a repair.

Kijima and Sumita [15] described an imperfect repair model by introducing the concept of “virtual age” for repairable products. If a product has the virtual age V_{i-1} immediately after the $(i-1)$th repair occurs, then the time to the ith product failure, X_i, exhibits the following conditional cumulative distribution function:

$$F(x_i) = \Pr\{X_i \le x_i | V_{i-1} = v_{i-1}\} = \frac{F(x_i + v_{i-1}) - F(v_{i-1})}{1 - F(v_{i-1})} , \tag{1}$$

where $F(x)$ is the Cumulative Distribution Function (CDF) of the time to the first failure of a new product. Under the assumption that the time to first failure follows a Weibull distribution, the conditional CDF of the GRP model is defined as follows:

$$F(x_i) = 1 - \exp\left[\left(\frac{v_{i-1}}{\alpha}\right)^{\beta} - \left(\frac{x_i + v_{i-1}}{\alpha}\right)^{\beta}\right] . \tag{2}$$

and the corresponding Probability Density Function (PDF) is:

$$f(x_i) = \frac{\beta}{\alpha^{\beta}}(x_i + v_{i-1})^{\beta-1}\exp\left\{\frac{1}{\alpha^{\beta}}[v_{i-1}^{\beta} - (x_i + v_{i-1})^{\beta}]\right\} , \tag{3}$$

where α and β are the scale parameter and shape parameter of the Weibull distribution, respectively. Under these assumptions and notations, Kijima [16] proposed two models that are referred to as the GRP I and GRP II models.

2.1. The GRP I model

The GRP I model assumes that the ith repair only compensates for the damage incurred during the time between the $(i-1)$th and the ith failure. The model reduces the additional age x_i to qx_i, where q is the repair level of each repair and $0 \le q \le 1$. For simplification, the repair level is treated as a constant. Under this assumption, the virtual age of the product after the ith repair is:

$$v_i = v_{i-1} + qx_i = q\sum_{j=1}^{i} x_j = qt_i , \tag{4}$$

where $v_0 = 0$. The conditional Weibull CDF in the GRP I model is:

$$F(x_i) = 1 - \exp\left[\left(\frac{qt_{i-1}}{\alpha}\right)^{\beta} - \left(\frac{x_i + qt_{i-1}}{\alpha}\right)^{\beta}\right] . \tag{5}$$