Accepted Manuscript

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PII: S0893-9659(17)30382-8
DOI: https://doi.org/10.1016/j.aml.2017.12.013
Reference: AML 5399

To appear in: *Applied Mathematics Letters*

Received date : 7 October 2017
Revised date : 11 December 2017
Accepted date : 12 December 2017

Please cite this article as: C. Liu, F. Wang, Y. Gu, Trefftz energy method for solving the Cauchy problem of the Laplace equation, Appl. Math. Lett. (2017), https://doi.org/10.1016/j.aml.2017.12.013

Trefftz energy method for solving the Cauchy problem of the Laplace equation

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Abstract

The inverse Cauchy problem of Laplace equation is hard to solve numerically, since it is highly ill-posed in the Hadamard sense. With this in mind, we propose a natural energy regularization technique to overcome the difficulty. In the linear space of the Trefftz bases for solving the Laplace equation, we introduce a novel concept to construct the Trefftz energy bases used in the numerical solution for the inverse Cauchy problem of the Laplace equation in arbitrary star-like plane domain. The Trefftz energy bases not only satisfy the Laplace equation but also preserve the energy, whose performance is better than the original Trefftz bases. We test the new method by two numerical examples.

Keywords: Trefftz energy bases method (TEBM), Laplace equation, Inverse Cauchy problem

1. Introduction

The numerical methods to solve linear partial differential equations (PDEs) by using a complete set of trial functions as the bases of numerical solutions can be divided into three types: (a) boundary type methods, of which the bases automatically satisfy the PDE, (b) domain type methods, of which the bases automatically satisfy all boundary conditions, and (c) the Kansa type methods, of which none of the above two requirements is satisfied.

It is known that for solving the Laplace equation in two-dimensional arbitrary bounded domain the set

$$\{1, r^k \cos k\theta, r^k \sin k\theta,\ k = 1, 2, \ldots\} \tag{1}$$

forms the T-complete functions, and the numerical solution expanded in terms of these bases is known as the Trefftz method [1, 2]. The Trefftz method is designed to satisfy the governing equation and leaves the unknown coefficients determined by satisfying the boundary conditions through the collocation, the least square or the Galerkin method, etc. [3, 4]. Because we only consider the inverse Cauchy problem in the interior domain, the singular bases like that $r^{-k} \cos k\theta$ and $r^{-k} \sin k\theta$ are not employed in this study.

The inverse Cauchy problem for the Laplace equation arises in a number of applications, such as the vibrations of structures, the non-destructive evaluations, the detection of corrosion inside