Accepted Manuscript

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PII: S0893-9659(17)30386-5
DOI: https://doi.org/10.1016/j.aml.2017.12.017
Reference: AML 5403

To appear in: *Applied Mathematics Letters*

Received date : 30 October 2017
Revised date : 20 December 2017
Accepted date : 20 December 2017

On the entropy method and exponential convergence to equilibrium for a recombination-drift-diffusion system with self-consistent potential

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Abstract

We consider a Shockley–Read–Hall recombination-drift-diffusion model coupled to Poisson's equation and subject to boundary conditions, which imply conservation of the total charge. As main result, we derive an explicit functional inequality between relative entropy and entropy production rate, which implies exponential convergence to equilibrium with explicit constant and rate. We report that the key entropy-entropy production inequality ought rather not to be formulated on the states space of the parabolic-elliptic system, but on the reduced states space of the associated nonlocal drift-diffusion problem, where the Poisson equation is replaced by the corresponding Green function.

Keywords: Semiconductor model, Shockley–Read–Hall recombination, drift-diffusion systems, self-consistent potential, entropy method, convergence to equilibrium
2010 MSC: 35B40, 82D37, 35K57

1. Introduction

This paper investigates the large-time-behaviour of the following recombination-drift-diffusion-Poisson system on a bounded Lipschitzian domain $\Omega \subset \mathbb{R}^m$:

$$\begin{cases} \partial_t n = \nabla \cdot J_n(n,\psi) - R(n,p), & J_n := \mu_n(\nabla n + n\nabla(\psi + V_n)), \\ \partial_t p = \nabla \cdot J_p(p,\psi) - R(n,p), & J_p := \mu_p(\nabla p + p\nabla(-\psi + V_p)), \\ -\varepsilon\,\Delta\psi = n - p - C, & \end{cases} \tag{1}$$

where n, p and ψ represent the concentrations of electrons and holes as well as the self-consistent electric potential. The recombination terms are of Shockley–Read–Hall form, i.e.

$$R := F(n,p,x)\Big(np - e^{-V_n - V_p}\Big), \qquad 0 < C_{\mathrm{F}} \le F(n,p,x), \tag{2}$$

and $V_n, V_p \in H^1(\Omega) \cap L^\infty(\Omega)$ are external potentials. The strictly positive functions μ_n and μ_p with $\mu_n, \mu_p \ge \underline{\mu} > 0$ a.e. in Ω denote scaled mobilities of electrons and holes under the assumption that the Einstein relations hold true, see e.g. [MRS90]. In addition, $C \in L^\infty(\Omega)$ describes the internal doping profile and $\varepsilon > 0$ is the permittivity constant. Note that the drift-diffusion fluxes J_n and J_p can also be written in terms of the quasi-Fermi potentials Φ_n and Φ_p, i.e.

$$J_n = \mu_n n \nabla\Phi_n, \quad \Phi_n := \psi + V_n + \ln n, \qquad J_p = \mu_p p \nabla\Phi_p, \quad \Phi_p := -\psi + V_p + \ln p.$$

The main aim of the paper is to prove exponential convergence to an equilibrium state $(n_\infty, p_\infty, \psi_\infty)$ of (suitable) solutions to (1)–(2) via the so-called entropy method. In our context, the entropy method aims to quantify the decay of the non-negative relative entropy functional

$$E(n,p,\psi) = \int_\Omega \Big(n \ln\frac{n}{n_\infty} - (n - n_\infty) + p\ln\frac{p}{p_\infty} - (p - p_\infty)\Big)\,dx + \frac{\varepsilon}{2}\int_\Omega |\nabla(\psi - \psi_\infty)|^2\,dx \ge 0 \tag{3}$$

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