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Numerical analysis for electroosmotic flow of fractional Maxwell fluids

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Abstract

The electroosmotic flow of fractional Maxwell fluid in a rectangular microchannel is studied in this paper. By means of the fractional Maxwell constitutive equation, and based on the experimental data, the nonlinear conjugate gradient method is proposed to get the viscoelastic parameters. Combined with the continuity equations and Cauchy momentum equations, the governing equations of velocity distribution are established. Besides, a fully discrete spectral method based on a difference method in the temporal direction and a Legendre spectral method in the spatial direction is introduced to solve the dimensionless governing equations. Finally, some results are presented to show the effectiveness of the proposed method.

Keywords:
Electroosmotic flow, Fractional Maxwell fluid, Nonlinear conjugate gradient method, Legendre spectral method

1. Introduction

Due to the potential applications as a tool for fundamental physical and biochemical processes, non-Newtonian fluid and electroosmosis have become the relatively important research areas in microfluidics [1–8]. Many researchers have concentrated on the study of non-Newtonian fluid and electroosmotic flow. An improved singular boundary method was applied for the numerical analysis of two-dimensional Stokes flow by Qu et al. [9]. Das and Chakraborty [10, 11] firstly studied the non-Newtonian effects to electroosmotic flow. Park and Lee [12] used a finite volume method to calculate the flow of the full Phan-Thien-Tanner model in a rectangular microchannel. Kaushik et al. [13] considered the start-up flow of a viscoelastic fluid in a microfluidic channel, by considering the Oldroyd-B constitutive model. AC electroosmotic flow of generalized Maxwell fluid in a rectangular mcrochannel was studied by Jian et al. [14]. Exact solutions for electroosmotic flow of viscoelastic fluids in rectangular microchannels was introduced by Zhao et al. [15]. Recently, constitutive equations with fractional and fractal derivatives have been proved to be useful for describing the non-Newtonian fluid and electroosmotic. Su et al. [16, 17] considered the rheological behaviors of non-Newtonian fluid based on the fractal derivatives. Wang et al. [18] investigated the analytical and numerical solutions of electroosmotic slip flows of fractional second grade fluids. Wang et al. [19] discussed the transient electroosmotic flow of a generalized fractional Maxwell fluid in a straight pipe with a circular cross-section.

In this paper, we consider the electroosmotic flow in a rectangular microchannel with the fractional Maxwell constitutive equation [20],

$$\sigma(t) + \tau^{\alpha-\beta}\frac{\mathrm{d}^{\alpha-\beta}\sigma(t)}{\mathrm{d}t^{\alpha-\beta}} = E\tau^{\alpha}\frac{\mathrm{d}^{\alpha}\gamma(t)}{\mathrm{d}t^{\alpha}}, \tag{1}$$

where $\sigma(t)$ is the shear stress, $\gamma(t)$ is the shear strain, $\tau = \eta/E$ is a relaxation time, in which E is a shear modulus, η is a viscosity constant. Also, α and β are Caputo's fractional derivative [21] parameters satisfying $0 \le \beta \le \alpha \le 1$.

The applications related to non-Newtonian fluid flow through microchannels are mostly associated with the transport of biofluids, with blood as one of the most common example [3]. But to the best of our knowledge, there is little

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