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Simple nonconforming brick element for 3D Stokes equations

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Abstract

A simple nonconforming brick element is proposed for 3D Stokes equations. This element has 15 degrees of freedom and reaches the lowest approximation order. In the mixed scheme for Stokes equations, we adopt our new element to approximate the velocity, along with the discontinuous piecewise constant element for the pressure. The stability of this scheme is proved and thus the optimal convergence rate is achieved. A numerical example verifies our theoretical analysis.

Keywords: 3D Stokes equations, nonconforming finite element, stability
2010 MSC: 65N30, 76M10

1. Introduction

The Stokes problem is a fundamental problem in fluid mechanics. Lower order nonconforming mixed finite element methods, such as the CR-P_0 element by Crouzeix and Raviart [2] for simplicial meshes and Rannacher and Turek's nonconforming rotated Q_1 (NR)-P_0 element for quadrilateral and hexahedral meshes [10], are particularly prefered due to their simplicity and effectivity. As a subspace method of the NR element, the quadrilateral nonconforming P_1 element for second order elliptic problems was proposed by Park and Sheen [8] in 2003. Shortly thereafter, Hu and Shi [4] presented the constrained NR (CNR) element, which interprets the Park-Sheen element in a parametric manner. Unfortunately, these two elements cannot be directly designed for Stokes equations with piecewise constant pressure since the stability conditions are not satisfied. Subsequent researches such as [5, 6, 9, 7] are dedicated to this issue. However, most of these contributions have a strict requirement for the partition and furthermore, they all deal with the 2D case.

In this note, we provide a simple nonconforming brick element of 15 degrees of freedom (DoFs) for 3D Stokes equations. The structures of this element and the associated global finite element spaces are provided (Section 2). We adopt this element to approximate the velocity and utilize the piecewise constant element for the pressure. The stability assertion is proved and then the lowest order of convergence rate is derived (Section 3). A numerical example is also given (Section 4). This work follows the standard notations in Sobolev spaces. The subscript Ω will be omitted in the expressions of norms, semi-norms and inner products. The notation $P_k(D)$ denotes the usual polynomial space over a domain D of degree no more than k. Further, the positive constant C might be different in different places.

2. A simple nonconforming brick element

Let $\widehat{Q} = [-1,1]^3$ be the reference cube. The vertices of $\widehat{Q}$ are given by $\widehat{V}_1 = (-1,-1,-1)^T$, $\widehat{V}_2 = (1,-1,-1)^T$, $\widehat{V}_3 = (1,1,-1)^T$, $\widehat{V}_4 = (-1,1,-1)^T$, $\widehat{V}_5 = (-1,-1,1)^T$, $\widehat{V}_6 = (1,-1,1)^T$, $\widehat{V}_7 = (1,1,1)^T$ and $\widehat{V}_8 = (-1,1,1)^T$. We also denote the faces of $\widehat{Q}$ by setting $\widehat{F}_1 = \widehat{V}_1\widehat{V}_4\widehat{V}_8\widehat{V}_5$, $\widehat{F}_2 = \widehat{V}_2\widehat{V}_3\widehat{V}_7\widehat{V}_6$, $\widehat{F}_3 = \widehat{V}_1\widehat{V}_2\widehat{V}_6\widehat{V}_5$, $\widehat{F}_4 = \widehat{V}_4\widehat{V}_3\widehat{V}_7\widehat{V}_8$, $\widehat{F}_5 = \widehat{V}_1\widehat{V}_2\widehat{V}_3\widehat{V}_4$ and $\widehat{F}_6 = \widehat{V}_5\widehat{V}_6\widehat{V}_7\widehat{V}_8$. See Figure 1.

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