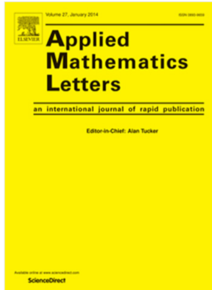

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Optimal Error Estimates for the Pseudostress Formulation of the Navier-Stokes Equations

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Abstract

In this paper, we prove optimal a priori error estimates for the pseudostress-velocity mixed finite element formulation of the incompressible Navier-Stokes equations, thus improve the result of Cai et al. [SINUM 48,(2010)]. This is achieved by applying Petrov-Galerkin type Brezzi-Rappaz-Raviart theory.

AMS(MOS) subject classification: 65N15, 65N30, 65N50, 76D05
Key words: Error estimates, pseudostress-velocity formulation, Navier-Stokes equations

1 Introduction

In this paper, we consider the pseudostress-velocity formulation of the stationary, incompressible Navier-Stokes equations (NSE). It is well known that as the viscosity varies along an interval, each solution of the NSE describes an isolated branch. This situation is expressed mathematically by the notion of branches of nonsingular solutions. The abstract framework of Brezzi, Rappaz, and Raviart (BRR) [2] is designed for the approximation of branches of nonsingular solutions for a class of nonlinear problem and provides effective machinery for unique solvability and convergence analysis [8, 5]. A recent application of the BRR framework can be found in [9], where a priori and a posteriori error estimates for mixed methods for scalar elliptic problems with gradient nonlinearities are derived.

The pseudostress-velocity formulation of the Stokes equations appears in the literature (see, for example, [1, 3, 6, 7]). This formulation allows to use Raviart-Thomas mixed finite elements of index $k \geq 0$ and is extended to approximate the NSE in [4]. There, it is shown that the discrete problem has a branch of nonsingular solutions, and the error bound $O(h^{k+1-\frac{d}{6}})$ is obtained in the $L^3(\Omega)^{d\times d} \times L^3(\Omega)^d$ ($d = 2$ or 3) norms, for sufficiently small h.

The main contribution of this paper is to obtain the optimal error bound $O(h^{k+1})$ in the $L^3(\Omega)^{d\times d} \times L^6(\Omega)^d$ norms for the pseudostress-velocity mixed finite element approximation of the NSE, which improves the existing result [4]. This is made possible by applying Petrov-Galerkin type BRR theory [5] rather than the standard BRR theory [2]. We note that Petrov-Galerkin type framework may provide a strategy to enhance error estimates, as illustrated by the example considered in this paper.

The remainder of this paper is organized as follows. In the next section, we introduce approximation of branches of nonsingular solutions based on the BRR theory. Section 3 is devoted to deriving the optimal a priori error estimates in the L^3-norm.

We finish this section introducing various spaces. Assume that Ω is a bounded, open, connected subset of $\mathbb{R}^d$ ($d = 2$ or 3) with Lipschitz continuous boundary $\partial\Omega$. We use the standard notations and definitions for the Sobolev spaces $W^{s,p}(\Omega)$ and $W^{s,p}(\partial\Omega)$ for $s \geq 0$ and $p \in [1,\infty]$ with associated norms denoted by $\|\cdot\|_{s,p} := \|\cdot\|_{s,p,\Omega}$ and $\|\cdot\|_{s,p,\partial\Omega}$. For $s = 0$, $W^{s,p}(\Omega)$ coincides with $L^p(\Omega)$. Moreover, the space $W^{s,2}(\Omega)$ will be written in the form $H^s(\Omega)$. Let

$$H(div;\Omega) := \{\boldsymbol{v} \in L^2(\Omega)^d \,|\, \nabla\cdot\boldsymbol{v} \in L^2(\Omega)\}, \quad W^{0,3}(div;\Omega) := \{\boldsymbol{v} \in L^3(\Omega)^d \,|\, \nabla\cdot\boldsymbol{v} \in L^2(\Omega)\},$$
$$L^2_0(\Omega) := \left\{ q \in L^2(\Omega) \,\middle|\, \int_\Omega q\,dx = 0 \right\}.$$

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