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Global strong solutions to the 3D full compressible Navier–Stokes system with vacuum in a bounded domain

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ABSTRACT

In this short paper we establish the global well-posedness of strong solutions to the 3D full compressible Navier–Stokes system with vacuum in a bounded domain $\Omega \subset \mathbb{R}^3$ by the bootstrap argument provided that the viscosity coefficients λ and μ satisfy that $7\lambda > 9\mu$ and the initial data ρ_0 and u_0 satisfy that $\|\rho_0\|_{L^\infty(\Omega)}$ and $\|\rho_0|u_0|^5\|_{L^1(\Omega)}$ are sufficiently small.

1. Introduction

In this short paper, we consider the following initial and boundary problem to the 3D full compressible Navier–Stokes system in a bounded domain $\Omega \subset \mathbb{R}^3$:

$$\partial_t \rho + \operatorname{div}(\rho u) = 0 \quad \text{on} \quad \Omega \times (0,\infty), \tag{1.1}$$

$$\partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) - \mu \Delta u - (\lambda+\mu)\nabla \operatorname{div} u + \nabla p = 0 \quad \text{on} \quad \Omega \times (0,\infty), \tag{1.2}$$

$$C_V\{\partial_t(\rho\theta) + \operatorname{div}(\rho u \theta)\} - \Delta\theta + p \operatorname{div} u = \frac{\mu}{2}|\nabla u + \nabla u^t|^2 + \lambda(\operatorname{div} u)^2 \quad \text{on} \quad \Omega \times (0,\infty), \tag{1.3}$$

$$u = 0, \ \frac{\partial\theta}{\partial n} = 0 \quad \text{on} \quad \partial\Omega \times (0,\infty), \tag{1.4}$$

$$(\rho, \rho u, \rho\theta)(\cdot, 0) = (\rho_0, \rho_0 u_0, \rho_0\theta_0) \quad \text{in} \quad \Omega. \tag{1.5}$$

Here the unknowns ρ, u, and θ denote the density, velocity and temperature of the fluid, respectively. The pressure $p := R\rho\theta$ and the internal energy $e := C_V\theta$ with positive constants R and C_V. λ and μ are two

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viscosity constants satisfying $\mu > 0$ and $\lambda + \frac{2}{3}\mu \geq 0$. n is the unit outward normal vector to the smooth boundary $\partial\Omega$ of Ω.

If the initial density ρ_0 has a positive lower bound, the global existence of small smooth solutions to the problem (1.1)–(1.5) was obtained in [1,2] three decades ago.

If the initial data may contain vacuum, Cho and Kim [3] proved the local well-posedness of strong solutions to the problem (1.1)–(1.5) under the compatibility conditions:

$$-\mu\Delta u_0 - (\lambda+\mu)\nabla \mathrm{div}\, u_0 + R\nabla(\rho_0\theta_0) = \sqrt{\rho_0}g_1, \tag{1.6}$$

$$\Delta\theta_0 + \frac{\mu}{2}|\nabla u_0 + \nabla u_0^t|^2 + \lambda(\mathrm{div}\, u_0)^2 = \sqrt{\rho_0}g_2, \tag{1.7}$$

with $g_1, g_2 \in L^2(\Omega)$.

Recently, Huang and Li [4] prove that the global well-posedness of strong solutions to the full compressible Navier–Stokes equations in the whole space $\mathbb{R}^3$ with smooth initial data which are of small energy but possibly large oscillations where the initial density is allowed to vanish, see also [5]. However, the methods developed in [4,5] cannot be applied directly to bounded domain case.

The aim of this paper is to prove that, although the initial density may contain vacuum, the problem (1.1)–(1.5) still has a unique global strong solution for small initial data. Our results read as

Theorem 1.1. *Let $0 \leq \rho_0 \in W^{1,6}(\Omega), u_0 \in H_0^1(\Omega) \cap H^2(\Omega), 0 \leq \theta_0 \in H^2(\Omega)$ with $\frac{\partial\theta_0}{\partial n} = 0$ on $\partial\Omega$ and (1.6), (1.7) hold true. If*

$$7\lambda > 9\mu \text{ and } \|\rho_0\|_{L^\infty} + \|\rho_0|u_0|^5\|_{L^1} \tag{1.8}$$

is sufficient small, then the problem (1.1)–(1.5) has a unique global-in-time strong solution.

Remark 1.1. It is interesting to note that the initial temperature θ_0 need not be small in our results.

Remark 1.2. It is possible to establish a similar result for the full compressible magnetohydrodynamical system.

Remark 1.3. When $\Omega := \mathbb{R}^3$ and consider the isentropic Navier–Stokes system, a similar result can be proved when $\|\rho_0\|_{L^p}$ is small for some large p by the method developed here and a blow-up criterion

$$\rho \in L^p(\mathbb{R}^3 \times (0,T)) \tag{1.9}$$

proved in [6].

Remark 1.4. A similar result holds true when the boundary condition $\frac{\partial\theta}{\partial n} = 0$ on $\partial\Omega$ is replaced by $\theta = 0$ on $\partial\Omega$.

To prove Theorem 1.1, we will use the following abstract bootstrap argument or continuity argument ([7], Page 20).

Lemma 1.2 ([7]). *Let $T > 0$. Assume that two statements $C(t)$ and $H(t)$ with $t \in [0,T]$ satisfy the following conditions:*

(a) If $H(t)$ holds for some $t \in [0,T]$, then $C(t)$ holds for the same t;
(b) If $C(t)$ holds for some $t_0 \in [0,T]$, then $H(t)$ holds for t in a neighborhood of t_0;
(c) If $C(t)$ holds for $t_m \in [0,T]$ and $t_m \to t$, then $C(t)$ holds;
(d) $C(t)$ holds for at least one $t_1 \in [0,T]$.

Then $C(t)$ holds for all $t \in [0,T]$.