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Explicit decay rate for coupled string-beam system with localized frictional damping[☆]

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ABSTRACT

The large-time behavior of a 1-d coupled string-beam system is considered. Combining a detailed spectral analysis with resolvent estimation, we obtain two kinds of energy decay rates of the string-beam system with different locations of the frictional damping. On one hand, if the frictional damping is only actuated in the beam part, the system lacks exponential decay. Specifically, the optimal polynomial decay rate t^{-1} is obtained under smooth initial conditions. On the other hand, if the frictional damping is only effective in the string part, the exponential decay of energy is presented. Some numerical simulations are given to support these results.

1. Introduction

The elastic multi-link structures have many applications in engineering, such as aircrafts, satellite antennas and so on (see [1]). One kind of such structures consists of elastic strings and beams. The large-time behavior of this kind of structure with different damping has been studied a lot in recent years. For instance, Nicaise and Valein in [2] showed the exponential stability of wave equation on 1-d networks with delay terms in the nodal damping. Zhang and Zuazua in [3] obtained the optimal polynomial decay rate for a wave-heat system based on a detailed spectral analysis. Han and Xu in [4] proved the exponential decay of the energy for star-shaped networks of elastic beams. More related references can be found in [5–7] and the references therein. To some extent, the large-time behavior of the multi-link systems is dependent on the locations of the damping in the systems. Rivera et al. in [8] studied the transmission problems for elastic strings with localized frictional and Kelvin–Voigt dissipation. They found that the energy decay rate of this system depends on the location of the Kelvin–Voigt damping. Ammari et al. in [9,10] and [11] considered the nodal

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feedback stabilization for networks of strings and beams. They obtained that the decay rate of a closed-loop system depends on the positions of the nodal feedback controllers. However, there are still very few literatures on such a study. This motivates us to study the explicit energy rates of a coupled string-beam system with different internalized frictional damping.

We consider the following PDEs' system composed of a string and a beam coupled at the interface point, while the frictional damping is actuated either in the domain of string part or beam part. More precisely, the system can be described as follows:

$$\begin{cases} y_{tt}(x,t) - y_{xx}(x,t) + \alpha y_t(x,t) = 0, \ 0 < x < 1, \ t > 0, \\ \theta_{tt}(x,t) + \theta_{xxxx}(x,t) + \gamma\theta_t(x,t) = 0, \ 1 < x < 2, \ t > 0, \\ y(0,t) = 0, \ \theta(2,t) = \theta_{xx}(2,t) = 0, \ \theta_{xx}(1,t) = 0, \ t > 0, \\ y(1,t) = \theta(1,t), \ y_x(1,t) + \theta_{xxx}(1,t) = 0, \ t > 0, \\ y(x,0) = y_0(x), \ y_t(x,0) = y_1(x), \ \theta(x,0) = \theta_0(x), \ \theta_t(x,0) = \theta_1(x), \end{cases} \tag{1}$$

where $y(x,t)$ and $\theta(x,t)$ are the displacement of the string and beam at time t and position x, respectively. $\alpha, \ \gamma$ are constants satisfying $\alpha\gamma = 0$ and $\alpha + \gamma > 0$. The energy functional for the system above is given as $E(t) = \int_0^1 [|y_x|^2 + |y_t|^2 + |\theta_{xx}|^2 + |\theta_t|^2]dx$, and its derivative $E'(t) = -\alpha\int_0^1 |y_t|^2 dx - \gamma\int_0^1 |\theta_t|^2 dx \leq 0$. So the energy of the system above is decreasing. It should be noted that if $\alpha, \ \gamma$ are both positive (that is to say, the frictional damping is effective in the global domain), it is easy to check that the solution to the system decays to zero exponentially by the energy multiplier or frequency domain method. However, it is unknown whether the exponential decay still holds if either α or γ is null. In this work, we give a detailed spectral analysis for this system and focus on presenting a complete analysis for the large-time behavior of the solution to system (1). We conclude that if the frictional damping is effective in the domain of the beam part, the system lacks exponential decay, while if it is effective in the domain of the string part, the exponential decay can be obtained. Note that this result is consistent with those regarding the nodal damping on string-beam system given in [9]. However, the authors in that paper did not discuss the optimality of polynomial decay under smooth initial conditions, which will be further discussed in our work.

The optimality of polynomial decay rate is a tough issue to discuss, which is also the main contribution of this paper. In general, it can be determined by the relationship between the real and imaginary parts of the spectra of the corresponding system operator, in other words, the speed of the spectra approaching the imaginary axis. However, it is difficult to get explicit expressions of the spectra due to the couplings between the string and beam in the system. In this work, using some special asymptotic techniques, we obtain an explicit expression of the spectra. Based on this expression, together with the resolvent estimation along the imaginary axis, it is enough to obtain the optimal polynomial decay rate for the system.

The paper is organized as follows. In Section 2, the system is described in an appropriate Hilbert space setting and the well-posedness is proved. Section 3 is devoted to discussing the large-time behavior for the case $\alpha = 0, \ \gamma > 0$. We show that the system lacks exponential decay. Specifically, the optimal polynomial decay rate is obtained with smooth initial conditions. In Section 4, we obtain the exponential decay of the system for the case $\alpha > 0, \ \gamma = 0$, similarly. In Section 5, some numerical examples are given.

2. Well-posedness

This section is devoted to showing the well-posedness of system (1). For convenience, set $u(x,t) = y(1-x,t), \ w(x,t) = \theta(x+1,t), \ 0 < x < 1, \ t > 0$. Then system (1) can be transformed as follows.

$$\begin{cases} u_{tt}(x,t) - u_{xx}(x,t) + \alpha u_t(x,t) = 0, \ 0 < x < 1, \ t > 0, \\ w_{tt}(x,t) + w_{xxxx}(x,t) + \gamma w_t(x,t) = 0, \ 0 < x < 1, \ t > 0, \\ u(1,t) = 0, \ w(1,t) = w_{xx}(1,t) = 0, \ w_{xx}(0,t) = 0, \ t > 0, \\ u(0,t) = w(0,t), \ u_x(0,t) = w_{xxx}(0,t), \ t > 0, \\ u(x,0) = y_0(1-x), \ u_t(x,0) = y_1(1-x), \ w(x,0) = \theta_0(x+1), \ w_t(x,0) = \theta_1(x+1). \end{cases} \tag{2}$$