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A conservative parallel difference method for 2-dimension diffusion equation

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Abstract

In this paper, a conservative parallel difference scheme, which is based on domain decomposition method, for 2-dimension diffusion equation is proposed. In the construction of this scheme, we use the numerical solution on the previous time step to give a weighted approximation of the numerical flux. Then the sub-problems with Neumann boundary are computed by fully implicit scheme. What's more, only local message communication is needed in the program. We use the method of discrete functional analysis to give the proof of the unconditional stability and second-order convergence accuracy. Some numerical tests are given to verify the theory results.

Keywords: parallel difference, diffusion equation, conservative, domain decomposition

1. Introduction

Time-dependent diffusion equations are widely used in many fields of science and engineering. To solve diffusion equations efficiently, the classic fully implicit method is usually a natural choice because of its unconditional stability [1]. However a global linear algebraic systems resulting from the implicit method must be solved at each time level which cannot be implemented directly and naturally on parallel computers. On the other hand, the classic explicit scheme can not be used in some applications due to its severe time step-length restriction though it has intrinsic parallelism. Note that conservation is also an important property for diffusion equation, so it is reasonable to devise a conservative scheme for it. In summary it is necessary to devise new schemes for diffusion equations satisfying the following properties: (i) unconditional stability; (ii) second-order accuracy in space; (iii) intrinsic parallelism; (iv) conservation.

Several early attempts are carried out in [2–8]. Especially in [9, 10] some conservative schemes are devised and numerical tests show that they have good stability and second-order accuracy, but no theoretical proof is given. In [11] a conservative parallel scheme for one-dimensional diffusion equation is devised and proved theoretically to be of unconditional stability and second-order accuracy. But, as we know, for any conservation parallel scheme of multi-dimensional diffusion problems there is few theoretical result up to now. For two-dimensional problems, another approach based on splitting technique is proposed in [12] to devise parallelization procedures, which are different from all the parallel difference schemes mentioned above.

In this work, we construct a new parallel scheme for diffusion equations by predicting a numerical flux on the interface with the value on previous time-level. Then the fully implicit scheme is carried out on the sub-domains. This method can easily be extended to high-dimensional problems (for example 2 dimension ones). Both numerical tests and theoretical analysis demonstrate that the scheme is conservative, unconditionally stable and has second-order accuracy.

This article is organized as follows. In section 2, a conservative parallel difference scheme is given for 2-dimension and we also list main results in this part. In section 3, both unconditional stability and convergence are demonstrated for 1-dimension problem. We extend the proof to 2-dimension in section 4. In section 5 some numerical tests are given to verify the theoretical results. We conclude the paper in section 6.

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