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Stationary distribution and extinction of a stochastic predator–prey model with distributed delay

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A B S T R A C T

In this paper, we focus on a stochastic predator–prey model with distributed delay. We first obtain the existence of a stationary distribution to the positive solutions by stochastic Lyapunov function method. Then we establish sufficient conditions for extinction of the predator population, that is, the prey population is survival and the predator population is extinct.

1. Introduction

Recently, infinite delay has been widely introduced into equations used in mathematical biology since the works of Volterra [1] to translate the cumulative effect of the past history of a system. Many authors (see e.g. [2,3]) studied the stability and bifurcation of predator–prey models with distributed delay. Especially, Chen et al. [3] developed the following predator–prey model with distributed delay

$$\begin{cases} \dfrac{dx}{dt} = b_1 x(1 - \dfrac{x}{K}) - a_{12}xy, \\ \dfrac{dy}{dt} = -b_2 y + a_{21} \displaystyle\int_{-\infty}^{t} K(t-s)x(s)y(s)ds, \end{cases} \tag{1.1}$$

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where x denotes the density of the prey, y is the density of the predator, b_1 and K are intrinsic growth rate and carrying capacity of the prey, respectively, b_2 represents the mortality rate of the predator, a_{12} is the per-capita rate of predation of the predator and a_{21} denotes the product of the per-capita rate of predation and the rate of conversing prey into predator. All parameters are assumed to be positive constants. The kernel $K : [0, \infty) \to [0, \infty)$ is a L^1-function, normalized such as

$$\int_0^\infty K(s)ds = 1.$$

Moreover, in the real world, population system is inevitably affected by the environmental white noise. Stochastic differential equation models play an important role in population dynamics as they can provide some additional degree of realism compared to their corresponding deterministic counterparts. Thus many authors have introduced random perturbations into deterministic models to reveal the effects of environmental white noise on the population dynamics (see e.g. [4,5]). Motivated by the referred works, in this paper we adopt the approach used by Imhof and Walcher [6], and assume that the environmental white noise is proportional to variables x and y. Then corresponding to system (1.1), we obtain the following stochastic model

$$\begin{cases} dx = [b_1 x(1 - \frac{x}{K}) - a_{12}xy]dt + \alpha_1 x dB_1(t), \\ dy = [-b_2 y + a_{21} \int_{-\infty}^t K(t-s)x(s)y(s)ds]dt + \alpha_2 y dB_2(t), \end{cases} \tag{1.2}$$

where $B_i(t)$ are mutually independent standard Brownian motions with $B_i(0) = 0$, $\alpha_i^2 > 0$ denote the intensities of the white noise, $i = 1, 2$.

In this paper, we mainly consider the weak kernel case for convenience, that is $K(t) = \sigma e^{-\sigma t}$ with $\sigma > 0$ which is initially introduced by MacDonald [7]. The strong kernel case can be investigated similarly. The weak kernel and the strong kernel have been extensively used in biological system, such as population systems [2,8] and epidemiology [9].

Setting

$$z(t) = \int_{-\infty}^t \sigma e^{-\sigma(t-s)} x(s)y(s)ds,$$

then by the linear chain technique, system (1.2) is transformed into the following equivalent system

$$\begin{cases} dx = [b_1 x(1 - \frac{x}{K}) - a_{12}xy]dt + \alpha_1 x dB_1(t), \\ dy = [-b_2 y + a_{21} z]dt + \alpha_2 y dB_2(t), \\ dz = \sigma(xy - z)dt. \end{cases} \tag{1.3}$$

In this paper, we mainly focus on establishing sharp sufficient criteria for the existence of a stationary distribution of system (1.3). As far as we know, there have been some other results on stationary distribution of stochastic predator–prey model with time delay, for example, stochastic delay cascade predator–prey model [10] and stochastic delay two-predator one-prey model [11]. However, these papers are mainly concerned with discrete delay while the present paper is concerned with distributed delay and the innovation of this paper is obvious.

Throughout this paper, unless otherwise specified, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (see [12]) and we also let $B_i(t)$ be defined on the complete probability space, $i = 1, 2$. Define $\mathbb{R}_+^d = \{x = (x_1, x_2, \ldots, x_d) \in \mathbb{R}^d : x_i > 0, 1 \leq i \leq d\}$. If f is a bounded function on $[0, \infty)$, define $f^u = \sup_{t\geq 0} f(t)$.

The following lemma concerns the existence and uniqueness of global positive solutions to system (1.3). Since the proof is standard and we omit it here.