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Abstract

The aim of this paper is to solve a class of non-autonomous linear fractional differential equations with random inputs. A mean square convergent series solution is constructed in the case that the fractional order α of that Caputo derivative lies in $]0, 1]$ using a random Fröbenius approach. The analysis is conducted by using the so-called mean square random calculus. The mean square convergence of the series solution is established assuming mild conditions on random inputs (diffusion coefficient and initial condition). We show that these conditions are satisfied for a variety of unbounded random variables. In addition, explicit expressions to approximate the mean, the variance and the covariance functions of the random series solution are given. Two full illustrative examples are shown.

Keywords: Random fractional differential equations, random mean square calculus.

1. Motivation and preliminaries

The combination of random/stochastic and fractional calculus is gaining influence in applied mathematics over the last few years through stochastic/random fractional differential equations (SFDEs/RFDEs). On the one hand, fractional calculus provides a powerful generalization of the classical derivative which is able to model memory and hereditary properties of various materials and processes, like viscoelasticity, phenomena with microscopic complex behaviour (fractals), etc., [1, 2, 3, 4, 5]. On the other hand, stochastic/random calculus is the natural framework to describe phenomena with inherent uncertainty usually meet in physics, biology, engineering, finance, etc. There are two main approaches when uncertainty is considered in fractional differential equations, namely, SFDEs and RFDEs. In the former case, uncertainty is usually modelled through a stochastic process, like Wiener process, having an irregular (e.g., continuous but nowhere differentiable) sample behaviour [6]. In this approach uncertainty is often restricted to specific probabilistic patterns (typically Gaussian, Poisson, Markovian, etc.). RFDEs are those in which random effects are directly manifested in input parameters (initial/boundary conditions, source terms, coefficients, etc.), which seems to be more natural, since in many models they have a physical interpretation susceptible to encapsulate some kind of uncertainty due to measurement errors and/or the inherent complexity of the phenomenon under analysis [7]. Another important advantage of RFDEs is that inputs can have a wide variety of probability distributions like Binomial, Poisson, Beta, Gamma, Gaussian, etc. In the extant literature, most of the contributions have focussed on SFDEs. Some recent contributions dealing with existence and uniqueness to solutions of RFDEs can be found in [8, 9]. These results extend their deterministic counterpart. The goal of this paper is to contribute to the emergent area of RFDEs by randomizing a class of non-autonomous fractional differential equations (see (1)) that has been studied, in its deterministic formulation, using the successive approximation method or Picard's method (see, [1, p.232]). As

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