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Thresholds and bistability in virus-immune dynamics☆

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Abstract

In this paper, we investigate an impaired or immunosuppressive HIV infection model. By analyzing the model, we obtain two thresholds R_0 and R_c. It is shown that R_0 determines whether the virus dies out. We also obtain the post-treatment control threshold and the elite control threshold. There exists a bistable interval between these two thresholds. When $R_0 > R_c$, immune intensity is in the bistable interval, which implies that the system has bistable behavior and as such the virus is under post-treatment control. On the other hand, when the immune intensity is greater than the elite immune control threshold, virus will be under post-treatment control. Our investigation shows that when antiviral therapy cannot completely clear the virus, introducing immunotherapy is an optimal choice.

Keywords: Virus-immune dynamics, Post-treatment control threshold, Elite control threshold, Bistability
2010 MSC: 34D20, 92D30

1. Introduction

In this paper, we investigate the impaired or immunosuppressive HIV infection model [1, 2],given by

$$\begin{aligned} \frac{dy(t)}{dt} &= ry(t)(1-\frac{y(t)}{K}) - ay(t) - py(t)z(t), \\ \frac{dz(t)}{dt} &= \frac{cy(t)z(t)}{1+\eta y(t)} - bz(t) - qy(t)z(t), \end{aligned} \tag{1}$$

where y is the density of virus, and z is the density of immune cells. All parameters in this model take non-negative values. Here, parameter r is the viral replication rate at low viral loads. Here, a is the viral elimination rate caused by natural decay and antiviral therapy-related decay. Virus can be killed by immune cells at rate pyz. Immune cells have death rate b and can be inhibited by virus at rate qyz. Here, c is the immune intensity, and $\frac{cyz}{1+\eta y}$ is the proliferation term.

Komarova et al. [1] constructed a virus-immune model and studied the boosting immunity by antiviral drug therapy. Their investigation revealed the relationship among timing, efficacy, and success. Shu et al. [2]showed that under some conditions, the virus-immune model display bistable behavior. Wang and Liu [3] designed a

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