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Breather wave, rogue wave and solitary wave solutions of a coupled nonlinear Schrödinger equation☆

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Abstract

Under investigation in this paper is a coupled nonlinear Schrödinger (NLS) equation, which describes nonlinear pulse propagation in optical fibers by retaining terms up to the next leading asymptotic order. Based on the Lax pair of the coupled NLS equation, we construct the determinant representation of the N-fold Darboux transformation(DT). Furthermore, by using the obtained N-fold DT, we obtain its higher-order soliton, breather and rogue wave solutions. Finally, the dynamic characteristics of these solutions are discussed.

Keywords: The coupled NLS equation, Darboux transformation, Breather wave, Rogue wave, Solitary wave.
2010 MSC: 35Q51, 35Q53, 35C99, 68W30, 74J35.

1. Introduction

In [1], Fokas proposed an integrable nonlinear Schrödinger (NLS) equation

$$iu_t - \nu u_{tx} + \gamma u_{xx} + \sigma|u|^2(u + i\nu u_x) = 0, \qquad \sigma \pm 1, \tag{1.1}$$

by using bi-Hamiltonian methods, where γ and ν are nonzero real parameters and $u(x,t)$ is a complex valued function. When $\nu = 0$, (1.1) reduces to the NLS equation. Eq. (1.1) arises as a model for nonlinear pulse propagation in monomode optical fibers and is the first negative member of the integrable hierarchy associated with the derivative NLS equation[2]. In [3], Lenells and Fokas applied the bi-Hamiltonian structure to write down the first few conservation laws of (1.1) and derive its Lax pair, by which they solve the initial value problem and analyze solitons.

Recently, rogue wave(RW), a special type of solitary waves, also known as monster waves, killer waves, extreme waves, gaint waves, has attracted a lot of attention in physical branch. Rogue wave was observed in many fields, such as oceanics [4]-[6] and nonlinear optics [7]-[9], and there are several techniques for investigating rogue wave such as dressing method, Bäcklund transformation, Darboux transformation (DT) method, bilinear method and so on [10]-[26]. Recently, we have studied the breather wave, rogue wave and solitary wave solutions of some nonlinear differential equations by using the Hirota bilinear method [27]-[32].

In this paper, we mainly study an integrable generalization of the coupled nonlinear Schrödinger equation equation[33]

$$\begin{aligned} &u_{xt} + \alpha\beta^2 u - 2i\alpha\beta u_x - \alpha u_{xx} + i\alpha\beta^2 uvu_x = 0,\\ &v_{xt} + \alpha\beta^2 v - 2i\alpha\beta v_x - \alpha v_{xx} - i\alpha\beta^2 uvv_x = 0, \end{aligned} \tag{1.2}$$

by using the Darboux transformation method. When $\alpha = 1, \beta = 1$, then Eq.(1.2) reduces to Fokas-Lenells system

$$\begin{aligned} &u_{xt} + u - 2iu_x - u_{xx} + iuvu_x = 0,\\ &v_{xt} + v + 2iv_x - v_{xx} - iuvv_x = 0, \end{aligned} \tag{1.3}$$

which has been studied in [34].

To the best of our knowledge, the breather and rogue wave solutions of Eq.(1.2) have not been investigated before. The primary purpose of the present paper is to employ Darboux transformation method to construct higher-order soliton, breather and rogue wave solutions of Eq.(1.2).

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