Accepted Manuscript

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PII: S0893-9659(17)30352-X
DOI: https://doi.org/10.1016/j.aml.2017.11.015
Reference: AML 5377

To appear in: *Applied Mathematics Letters*

Received date : 19 September 2017
Revised date : 19 November 2017
Accepted date : 19 November 2017

Please cite this article as: C. Qu, W. Zhou, B. Liang, Asymptotic behavior for a fourth-order parabolic equation modeling thin film growth, Appl. Math. Lett. (2017), https://doi.org/10.1016/j.aml.2017.11.015

Asymptotic behavior for a fourth-order parabolic equation modeling thin film growth *

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Abstract. The main goal of this work is to study a fourth-order parabolic equation with nonstandard growth conditions. For non-positive initial energy $J(u_0) \leq 0$, by using the concavity method, we establish that weak solutions blow up in finite time. On the other hand, we also derive the blow-up time estimate and the asymptotic behavior of weak solutions.

Keywords: fourth-order parabolic equation, nonstandard growth condition, asymptotic behavior

2010 Mathematics subject classification: Primary 35K58; 35K35; 35B40

1 Introduction

In this paper, we study the blow-up phenomenon of weak solutions to the following fourth-order parabolic equation which reads as follows,

$$\begin{cases} u_t + \Delta^2 u = u^{p(x)}, & (x,t) \in \Omega \times (0,T), \\ u = \Delta u = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^n$ is a bounded domain with smooth boundary, $u_0 \in H_0^2(\Omega)$ and $u_0 \not\equiv 0$. The term $\Delta^2 u$ describes the capillarity-driven surface diffusion. we assume that the function $p(x) : \Omega \mapsto (1, +\infty)$ satisfies that

$$1 < p^- \triangleq \inf_{x\in\Omega} p(x) \leq p(x) \leq p^+ \triangleq \sup_{x\in\Omega} p(x) < +\infty,$$

*Supported by the National Natural Science Foundation of China (Nos. 11401078, 11571062), the Fundamental Research Fund for the Central Universities (Nos. DC201502050402, DC201502050202), the Education Department Science Foundation of Liaoning Province of China (No. JDL2016029) and the Natural Science Fund of Liaoning Province of China (No. 20170540136)

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