Applied Mathematics Letters 77 (2018) 1–5

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Research Announcement

Inverse problem of potential theory

A.G. Ramm

Mathematics Department, Kansas State University, Manhattan, KS 66506-2602, USA

ARTICLE INFO

Article history:
Received 21 August 2017
Received in revised form 20 September 2017
Accepted 21 September 2017
Available online 2 October 2017

Keywords:
Inverse problems
Potential theory

ABSTRACT

P. Novikov in 1938 has proved that if $u_1(x) = u_2(x)$ for $|x| > R$, where $R > 0$ is a large number,

$$u_j(x) := \int_{D_j} g_0(x,y)dy, \quad g_0(x,y) := \frac{1}{4\pi|x-y|},$$

and $D_j \subset \mathbb{R}^3$, $j = 1, 2$, $D_j \subset B_R$, are bounded, connected, smooth domains, star-shaped with respect to a common point, then $D_1 = D_2$. Here $B_R := \{x : |x| \le R\}$.

Our basic results are:

(a) the removal of the assumption about star-shapeness of D_j,

(b) a new approach to the problem,

(c) the construction of counter-examples for a similar problem in which g_0 is replaced by $g = \frac{e^{ik|x-y|}}{4\pi|x-y|}$, where $k > 0$ is a fixed constant.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

Suppose there are two bodies D_j, $j = 1, 2$, uniformly charged with charge density 1. Let the corresponding potentials be $u_j(x) = \int_{D_j} \frac{dy}{4\pi|x-y|}$. Assume that $u_1(x) = u_2(x)$ for $|x| > R$, where $R > 0$ is a large number. The classical question (inverse problem of potential theory) is: *does this imply that $D_1 = D_2$?*

P. Novikov in 1938 (see [1]) has proved a uniqueness theorem for the solution of inverse problem (IP) of potential theory under a special assumption, see Proposition 1.

Let

$$u(x) = \int_D g_0(x,y)dy, \quad g_0(x,y) := \frac{1}{4\pi|x-y|}, \tag{1}$$

where $D \subset \mathbb{R}^3$ is a bounded, connected, C^2-smooth domain.

We use the following notations: D_j, $j = 1, 2$, are two different domains D, S_j is the boundary of D_j, $D'_j = \mathbb{R}^3 \setminus D_j$, S^2 is the unit sphere in $\mathbb{R}^3$, B_R is the ball of radius R, centered at the origin, $B'_R = \mathbb{R}^3 \setminus B_R$,

E-mail address: ramm@math.ksu.edu.

http://dx.doi.org/10.1016/j.aml.2017.09.012
0893-9659/© 2017 Elsevier Ltd. All rights reserved.

$D_j \subset B_R$, $D_{12} := D_1 \cup D_2$, $D'_{12} := \mathbb{R}^3 \setminus D_{12}$, $\mathcal{D} := D^{12} = D_1 \cap D_2$, $\mathcal{D}' = \mathbb{R}^3 \setminus \mathcal{D}$, $\mathcal{S}$ is the boundary of $\mathcal{D}$, and let $u_j(x) = \int_{D_j} g_0(x,y)dy$, $j = 1, 2$.

P. Novikov has proved the following result, see [1]:

Proposition 1. *If $u_1(x) = u_2(x)$ for $|x| > R$, then $D_1 = D_2$ provided that D_j, $j = 1, 2$, are star-shaped with respect to a common point.*

In [2] this result is generalized: the existence of the common point with respect to which D_1 and D_2 are star-shaped is not assumed, but D_j are still assumed star-shaped.

In [3], p. 334, a new proof of Proposition 1 was given. In [4–11], some inverse problems and symmetry problems are studied.

The goal of this paper is to give a new method for a proof of a generalization of Proposition 1. In this generalization

(a) The assumptions about star-shapeness of D_j, $j = 1, 2$, are discarded (Theorem 1);

(b) A new approach to the IP is developed;

(c) A similar inverse problem is studied in the case when $g_0(x,y)$ is replaced by the Green's function of the Helmholtz operator, $g(x,y) := \frac{e^{ik|x-y|}}{4\pi|x-y|}$, $k = const > 0$ is fixed, and u_j is replaced by

$$U_j(x) = \int_{D_j} g(x,y)dy.$$

The result is formulated in Theorem 2.

The idea of our proof does not use the basic idea of [1,2], or [3]. Our proof is based on some lemmas.

Lemma 1. *If $R(\phi)$ is the element of the rotation group in $\mathbb{R}^3$ then $\frac{\partial R(\phi)x}{\partial \phi}|_{\phi=0} = [\alpha, x]$, where $[x, y]$ is the cross product of two vectors, α is the unit vector around which the rotation by the angle ϕ takes place, and x is an arbitrary vector.*

This lemma is proved in [3], p. 416.

Lemma 2. *The set of restrictions on S of all harmonic in B_R functions is dense in $L^2(S)$, where S is the boundary of D.*

Proof of Lemma 2. Let us assume the contrary and derive a contradiction. Without loss of generality one may assume f to be real-valued. Suppose $f \not\equiv 0$ is orthogonal in $L^2(S)$ to any harmonic function h, that is,

$$\int_S fhds = 0 \tag{2}$$

for all harmonic functions in B_R.

Define

$$v(x) := \int_S g_0(x,s)fds.$$

Assumption (2) implies $v(x) = 0$ in $\mathbb{R}^3$. Indeed, there exists a unique solution to the problem

$$\Delta h = 0 \quad in \quad D, \quad h|_S = f.$$

For this h Eq. (2) implies that $f = 0$, so $v = 0$ in $\mathbb{R}^3$. $\square$

Download English Version:

https://daneshyari.com/en/article/8054102

Download Persian Version:

https://daneshyari.com/article/8054102

Daneshyari.com