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Generalized integral inequality: Application to time-delay systems

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Abstract

This paper investigates a stability problem for linear systems with time-delay. By constructing simple Lyapunov-Krasovskii functional (LKF), and utilizing a new generalized integral inequality (GII) proposed in this paper, a sufficient stability condition for the systems will be derived in terms of linear matrix inequalities (LMIs). Two illustrative examples are given to show the superiorities of the proposed criterion.

Keywords: Systems with time-delays, time-invariant, generalized integral inequality, Lyapunov method.

1. Introduction

Let us consider time-delay systems (TDSs) given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + A_D \int_{t-h}^{t} x(s)ds, \\ x(t) &= \phi(t), \ \ t \in [-h, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\phi(t)$ is an initial function, A, A_d and $A_D \in \mathbb{R}^{n\times n}$ are known constant matrices, and $h \in [h_m, h_M]$ is a time-constant delay.

Here, because it is well known that the time-delay causes poor performance or instability of systems, a great number of results on delay-dependent stability condition for TDSs have been reported in the literature. In this field, an important issue is to find less conservative conditions guaranteeing the asymptotic stability of TDSs. Here, the conditions are classified under two main perspectives that they are referred to Section 4 for further details. Naturally, various methods exist but the primary concern is a fundamental study on a new bound of the inequality for the integral of quadratic functions (IQFs). In this regards, many researchers have put their times and efforts on developments of IQFs such as the free matrix-based multiple integral inequality [1], Jensen integral inequality (JII) [2], Wirtinger-based integral inequality (WBI) [10], the different varieties of Wirtinger-based double integral inequality [5, 12, 13], Bessel-Legendre inequality (BLI) [11], and the auxiliary function-based integral inequalities (AFBIIs) [8, 9]. What all authors are noticing is how tightly bounded the inequality for IQFs is. Moreover, Cauchy-Schwartz-like inequality was addressed in the authors' works [6, 7] as summation case. Even though the aforementioned inequalities have different types of these, most of them can be derived from GII.

In line with this thinking, by applying Gram-Schmidt orthogonalization process and considering an weighted function unlike the work [9], GII is established in form of the inequality for the $k+1$ tuple IQFs given by $\int_a^b \int_{s_1}^b \cdots \int_{s_k}^b f(u)duds_k \cdots ds_1$, where $f(t)$ is any quadratic function. The main advantage of the

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