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EXISTENCE AND UNIQUENESS OF MILD SOLUTIONS TO THE MAGNETO-HYDRO-DYNAMIC EQUATIONS

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ABSTRACT. In this paper, we consider the incompressible magneto-hydro-dynamic equations in the whole space. We first show that there exists global mild solutions with small initial data in the scaling invariant space. The main technique we have used is implicit function theorem which yields necessarily continuous dependence of solutions for the initial data. Moreover, we gain the asymptotic stability of solutions as the time goes to infinity. Finally, as a byproduct of our construction of solutions in the weak L^p-spaces, the existence of self-similar solutions was established provided the initial data are small homogeneous functions.

Keywords Magneto-hydro-dynamic equations; Mild solutions; Implicit function theorem; Asymptotic stability; Self-similar solutions.

1. INTRODUCTION

In this paper, we mainly consider the following Magnetohydrodynamics (MHD) equations:

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{1}{Re}\Delta u + (u\cdot\nabla)u - S(b\cdot\nabla)b + \nabla P = 0 & in \quad \mathbb{R}^N\times(0,\infty), \\ \frac{\partial b}{\partial t} - \frac{1}{Rm}\Delta b + (u\cdot\nabla)b - (b\cdot\nabla)u = 0 & in \quad \mathbb{R}^N\times(0,\infty), \\ \operatorname{div} u = 0, \quad \operatorname{div} b = 0 & in \quad \mathbb{R}^N\times(0,\infty), \end{cases} \tag{1.1}$$

with initial data $u(x,0) = u_0(x), b(x,0) = b_0(x)$, where $u = u(x,t) = (u_i(x,t))_{i=1}^N$, $b = b(x,t) = (b_i(x,t))_{i=1}^N$, $P = P(x,t)$ denote the velocity of the fluid, the magnetic field and the pressure, respectively. For simplicity, let $Re = Rm = S = 1$. Such equations is the physical-mathematical framework that concerns the dynamics of magnetic fields in electrically conducting fluids [1].

The aim of this paper is to show the global existence of mild solutions to (1.1) in $\mathbb{R}^N(N \geq 2)$. Before we state our main results, here, we present some related work about MHD equations. Lions et al. [2] proved the existence and uniqueness of the global strong solutions of $2d$-MHD equations. They also proved the existence of the global weak solutions, existence and uniqueness of the locally strong solutions of $3d$-MHD equations. As a generalize result, in 2013, Tan et al. [3] defined a dissipation term $D(u,b)$ that stems from an eventual lack of smoothness in the solution of u and b, and then obtain a local equation of energy for weak solutions of $3d$-MHD equations. In particular, Huang et al. [5] proved that if the difference between the magnetic field and the velocity is small initially, which results in a global strong solution without the smallness restriction on the size of initial velocity or magnetic field, then it would keep small forever.

The other results of this paper are Theorem 1.2, and Corollary 1.1 about the global stability of mild solution to (1.1) under the small initial disturbance, and the forward self-similar solution. For the self-similar solution, Xin et al. [4] constructed a class of global unique forward self-similar solutions of $3d$ incompressible MHD equations, with small initial data which is being homogeneous of degree -1 and belonging to some Besov space, or the Lorentz space or pseudo-measure space. Here, we generalize their result about forward self-similar solution from 3-dimensional to N-dimensional with $N \geq 3$. We shall note that the method we used in the proof of Theorem 1.1-1.2 is provided by H. Kozono et al. in [6], the author consider the double chemotaxis model under the effect of the Navier-Stokes fluid and prove the existence of global mild solutions with the initial data in the scaling invariant space and clarify the asymptotic behavior of solutions as the time goes to infinity.

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