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An optimal reproducing kernel method for linear nonlocal boundary value problems

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Abstract

Based on the piecewise polynomial reproducing kernels, the authors presented the reproducing kernel method(RKM) for nonlocal boundary value problems. In this paper, we will present an optimal RKM for linear nonlocal boundary value problems by combining the piecewise polynomial kernel with polynomial kernel. The method is of higher accuracy. Also, the method can avoid reducing the inhomogeneous boundary conditions to homogeneous boundary conditions and constructing reproducing kernel satisfying corresponding homogeneous boundary conditions. Numerical examples are included to demonstrate the validity of the new method.

Keywords: Reproducing kernel method; Nonlocal boundary value problems; Polynomial kernel

1 Introduction

The applications of nonlocal boundary value problems have been found in a variety of different areas of applied mathematics and physics. The discussion on the existence and uniqueness of solutions to such problems can be found in [1-5]. The numerical methods for solving such problems have been discussed by some authors. Based on the piecewise polynomial reproducing kernels, some numerical methods have been proposed for some nonlocal boundary value problems [6-16]. Mehghan, Tatari and Saadatmandi [17-18] applied the Adomian decomposition method and Sinc-collocation method to multi-point boundary value problems. Tirmizi, Twizell and Islam [19] presented a second order method for solving third order three-point boundary value problems based on Pade approximation. Aziz, Siraj-ul-Islam and Nisar [20] introduced an efficient numerical algorithm based on Haar wavelet for solving a class of linear and nonlinear nonlocal boundary-value problems. Based on series representation via suitable base functions, Hao and Cong [21] proposed an efficient numerical algorithm for nonlocal boundary value problems.

In this paper, we will present an optimal RKM for the following nonlocal boundary value problems:

$$\begin{cases} u''(x)+p(x)u'(x)+q(x)u(x)=f(x), \ \ 0<x<1, \\ B_1(u)=\mu_1, \ \ B_2(u)=\mu_2, \end{cases} \tag{1.1}$$

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