Almost sure exponential stability sensitive to small time delay of stochastic neutral functional differential equations

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Almost Sure Exponential Stability Sensitive to Small Time Delay of Stochastic Neutral Functional Differential Equations

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Abstract

In this work, we establish a theory about the almost sure pathwise exponential stability property for a class of stochastic neutral functional differential equations by developing a semigroup scheme for the drift part of the systems under consideration and dealing with their pathwise stability through a perturbation approach, rather than through that one to get their moment stability first. As an illustration, we can show that some stochastic systems have their almost sure exponential stability not sensitive to small delays.

Keywords: Stability sensitive to small delays; Pathwise exponential stability; Stochastic Neutral functional differential equation.

Mathematics Subject Classifications: 60H15, 60G15, 60H05.

1. Introduction

Let H be a real Hilbert space with its norm $\|\cdot\|_H$ and inner product $\langle\cdot,\cdot\rangle_H$, respectively. We denote by $\mathscr{L}(H)$ the space of all bounded linear operators from H into itself. Let $r > 0$ be a constant and consider a deterministic time delay differential equation in H,

$$\begin{cases} dy(t) = \big[Ay(t) + By(t-r)\big]dt, \qquad t \ge 0,\\ y(0) = \phi_0 \in H,\ y(t) = \phi_1(t),\ t \in [-r, 0],\ \phi_1 \in L^2([-r,0], H), \end{cases} \tag{1.1}$$

where A is a linear operator generating a C_0-semigroup e^{tA}, $t \ge 0$, on H and B is some appropriate linear operator in H. Recall that the trivial solution of (1.1) is called exponentially stable if there exist number $M = M(\phi) \ge 1$ and constant $\gamma > 0$ such that $\|y(t)\|_H \le M(\phi)e^{-\gamma t}$ for all $t \ge 0$. It was observed by Datko et al. [3] (see also [2]) that small delays may destroy exponential stability of an infinite dimensional system like (1.1). More precisely, if the spectrum $\sigma(A)$ of A is unbounded along an imaginary line, it was shown (see Theorem 7.4, [1]) that one can find a bounded linear operator $B \in \mathscr{L}(H)$ such that $A + B$ generates an exponentially stable semigroup, i.e., $\|e^{t(A+B)}\| \le Ce^{-\beta t}$, $C, \beta > 0$, for all $t \ge 0$, and meanwhile for any $\varepsilon > 0$, there always exists $r \in (0, \varepsilon)$ such that the system (1.1) is not exponentially stable. From this observation, we can see that the unboundedness of the spectrum set of A along imaginary axes may cause trouble for exponential stability of (1.1). Thus, one need make additional assumptions on e^{tA}, $t \ge 0$, to obtain the desired stability. In fact, we have the following result whose proof is referred to [1].

Theorem 1.1. *Assume that A generates a norm continuous C_0-semigroup e^{tA}, $t \ge 0$, i.e., $e^{\cdot A} : [0, \infty) \to \mathscr{L}(H)$ is continuous and the semigroup generated by $A + B$, $B \in \mathscr{L}(H)$, is exponentially stable in H. Then there exists a constant $r_0 > 0$ such that the trivial solution of (1.1) is exponentially stable for all $r \in (0, r_0)$.*