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Representation of a Solution for a Fractional Linear System with Pure Delay☆

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Abstract

This paper gives a representation of a solution to the Cauchy problem for a fractional linear system with pure delay. We introduce the fractional delayed matrices cosine and sine of a polynomial of degree and establish some properties. Then, we use the variation of constants method to obtain the solution and our results extend those for second order linear system with pure delay. As an application, the representation of a solution is used to obtain a finite time stability result.

Keywords: Fractional linear system with pure delay, Fractional delayed matrices cosine and sine, Representation of solution, Finite time stability.

1. Introduction

Fractional differential equations arise naturally in real world phenomena; see [1, 2, 3, 4, 5] and the references therein. Khusainov et al. [6] obtained a representation of a solution to the Cauchy problem for a second order linear system with pure delay using the delayed matrix cosine of a polynomial of degree $\cos_\tau \Omega x$ and the delayed matrix sine of a polynomial of degree $\sin_\tau \Omega x$ (see [6, see Definitions 1 and 2]). For the literature on the representation and stability of solutions and the controllability of delay systems we refer the reader to [7, 8, 9, 10, 11, 12, 13, 14, 15].

Motivated by [6], we study a fractional linear system with pure delay of the form:

$$\begin{cases} {}^cD^\alpha_{-\tau^+}({}^cD^\alpha_{-\tau^+}y)(x) = -\Omega^2 y(x-\tau),\ y\in\mathbb{R}^n,\ \Omega\in\mathbb{R}^{n\times n},\ x\geq 0,\ \tau>0, \\ y(x)=\varphi(x),\ \dot{y}(x)=\dot{\varphi}(x),\ \ -\tau\leq x\leq 0, \end{cases} \tag{1}$$

where ${}^cD^\alpha_{-\tau^+}$ denotes the Caputo fractional derivative of order $\alpha\in(0,1]$ with the lower limit $-\tau$ (see Definition 2.1) and $\varphi\in C^1([-\tau,0],\mathbb{R}^n)$. We obtain the representation of the solution of (1) and discuss its finite time stability on $J=[0,T]$, $T>0$.

The rest of this paper is organized as follows. In section 2, we introduce the fractional delayed matrix cosine of a polynomial of degree $2k\alpha$: $\cos_{\tau,\alpha}\Omega x^\alpha$ and the fractional delayed matrix sine of a polynomial of degree $(2k+1)\alpha$: $\sin_{\tau,\alpha}\Omega x^\alpha$ (see Definitions 2.2 and 2.3). Then we give some properties of the Caputo derivative for $\cos_{\tau,\alpha}\Omega x^\alpha$ and $\sin_{\tau,\alpha}\Omega x^\alpha$ and give some norm estimations using the classical Mittag-Leffler function. We derive the formula of a solution of (1) using $\cos_{\tau,\alpha}\Omega x^\alpha$ and $\sin_{\tau,\alpha}\Omega x^\alpha$ and the variation of constants method. Finally we study finite time stability and give a sufficient condition.

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