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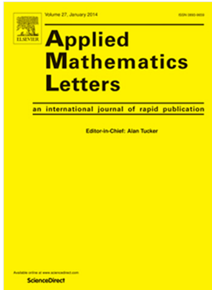

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On the convergence of the time reversal method for thermoacoustic tomography in elastic media

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Abstract

In this article, we consider the inverse source problem arising in thermo/photo-acoustic tomography in elastic media. We show that the time reversal method, proposed by Tittelfitz [Inverse Problems 28.5 (2012): 055004], converges with the sharp observation time without any constraint on the speeds of the longitudinal and shear waves.

1 Introduction

Let us consider the isotropic elastic wave propagation in the free space

$$\begin{cases} \mathbf{u}_{tt}(t,\mathrm{x}) - \Delta^* \mathbf{u}(t,\mathrm{x}) = 0, \ \mathrm{x} \in \mathbb{R}^3, \ t \geq 0, \\ \mathbf{u}(0,\mathrm{x}) = \mathbf{f}(\mathrm{x}), \ \mathbf{u}_t(0,\mathrm{x}) = 0, \ \mathrm{x} \in \mathbb{R}^3. \end{cases} \tag{1}$$

Here, $\mathbf{u} = (u_1, u_2, u_3)$ is the displacement vector,

$$\Delta^* \mathbf{u} = \nabla \Big[\mu(\mathrm{x}) \big(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \big) \Big] + \nabla (\lambda(\mathrm{x}) \, \nabla \cdot \mathbf{u}),$$

λ, μ are Lamé parameters, and $(\nabla \mathbf{u})_{i,j} = \frac{\partial \mathbf{u}_i}{\partial \mathrm{x}_j}$ is the Jacobian of $\mathbf{u}$ and $(\nabla \mathbf{u})^T$ is its transpose. We assume that $\lambda = \lambda(\mathrm{x}) \in C^\infty(\mathbb{R}^3)$ and $\mu = \mu(\mathrm{x}) \in C^\infty(\mathbb{R}^3)$ are nonnegative and bounded. Moreover, μ is bounded from below by a positive constant.

Let Ω be a bounded domain in $\mathbb{R}^3$ with the smooth boundary $\mathcal{S} = \partial\Omega$. In this article, assuming that $\mathrm{supp}(\mathbf{f}) \subset \Omega_0 \Subset \Omega$, we are interested in the following inverse source problem.

Problem 1.1. *Find the initial displacement* $\mathbf{f}$ *given the data* $\mathbf{g} = \mathbf{u}|_{[0,T]\times\mathcal{S}}$ *for some (observation time)* $T > 0$.

This problem arises in thermo/photo-acoustic tomography in elastic media. The same problem in the acoustic setting is very well-studied (see, e.g., [5, 11, 8, 16, 17, 14, 10]). Problem 1.1 was first studied in [21]. In that article, following the work of Stefanov and Uhlmann [16] for the acoustic setting, the author proposed a time reversal method to solve Problem 1.1. However, in order to prove the convergence of the method, the author assumed that the supremum of the P-wave speed is less than three times the infimum of the S-wave speed. Moreover, the required measurement time T has to be sufficiently large. In this article, we show that the same algorithm works without the restriction on the wave speeds. Moreover, the needed observation time T is the sharp observation time which comes from the visibility condition (see Assumption 2.2).

2 Notation and statement of the main result

Let us first introduction some notations. Let $U \subset \mathbb{R}^3$ be an open set and $\mathbf{f}, \mathbf{g} : U \to \mathbb{R}^3$. We define the following symmetric bilinear form

$$(\mathbf{f}, \mathbf{g})_{H(U)} = \int_U \lambda(\mathrm{x}) \, (\nabla \cdot \mathbf{f})(\nabla \cdot \mathbf{g}) + \frac{\mu(\mathrm{x})}{2} \left[\nabla \mathbf{f} + (\nabla \mathbf{f})^T \right] \cdot \left[\nabla \mathbf{g} + (\nabla \mathbf{g})^T \right] \, d\mathrm{x}$$

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