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# POSITIVE SOLUTIONS FOR SUPERDIFFUSIVE MIXED PROBLEMS

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Abstract. We study a semilinear parametric elliptic equation with superdiffusive reaction and mixed boundary conditions. Using variational methods, together with suitable truncation techniques, we prove a bifurcation-type theorem describing the nonexistence, existence and multiplicity of positive solutions.

## 1. Introduction

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a $C^2$-boundary $\partial\Omega$ and let $\Sigma_1, \Sigma_2 \subseteq \partial\Omega$ be two $(N-1)$-dimensional $C^2$-submanifolds of $\partial\Omega$ such that $\partial\Omega = \Sigma_1 \cup \Sigma_2$, $\Sigma_1 \cap \Sigma_2 = \emptyset$, $|\Sigma_1|_{N-1} \in (0, |\partial\Omega|_{N-1})$, and $\overline{\Sigma_1} \cap \overline{\Sigma_2} = \Gamma$. Here, $|\cdot|_{N-1}$ denotes the $(N-1)$-dimensional Hausdorff (surface) measure and $\Gamma \subset \partial\Omega$ is a $(N-2)$-dimensional $C^2$-submanifold of $\partial\Omega$.

In this paper, we study the following logistic-type elliptic problem:

$$(P_\lambda) \qquad \left\{ \begin{array}{ll} -\Delta u(z) = \lambda u(z)^{q-1} - f(z, u(z)) & \text{in } \Omega, \\ u|_{\Sigma_1} = 0, \ \left.\dfrac{\partial u}{\partial n}\right|_{\Sigma_2} = 0, \ u > 0, \ \lambda > 0. & \end{array} \right\}$$

When $f(z,x) = x^{r-1}$ with $r \in (2, 2^*)$, we get the classical logistic equation, which is important in biological models (see Gurtin & Mac Camy [8]). Depending on the value of $q > 1$, we distinguish three cases: (i) $1 < q < 2$ (subdiffusive logistic equation); (ii) $2 = q < r$ (equidiffusive logistic equation); (iii) $2 < q < r$ (superdiffusive logistic equation). In this paper, we deal with the third situation (superdiffusive case), which exhibits bifurcation-type phenomena for large values of the parameter $\lambda > 0$.

Let $E_{\Sigma_1} = \{u \in H^1(\Omega) : u|_{\Sigma_1} = 0\}$. This space is defined as the closure of $C_c^1(\Omega \cup \Sigma_1)$ with respect to the $H^1(\Omega)$-norm. Since $|\Sigma_1|_{N-1} > 0$, we know that for the space $E_{\Sigma_1}$, the Poincaré inequality holds (see Gasinski & Papageorgiou [7, Problem 1.139, p. 58]). So, $E_{\Sigma_1}$ is a Hilbert space equipped with the norm $||u|| = ||Du||_2$. Let $\mathcal{A} \in \mathcal{L}(E_{\Sigma_1}, E^*_{\Sigma_1})$ be defined by $\langle A(u), h\rangle = \int_\Omega (Du, Dh)_{\mathbb{R}^N} dz$ for all $u, h \in E_{\Sigma_1}$. We denote by $N_f$ the Nemitsky map associated with $f$, that is, $N_f(u)(\cdot) = f(\cdot, u(\cdot))$ for all $u \in E_{\Sigma_1}$.

The hypotheses on the perturbation term $f(z,x)$ are the following:

$H(f)$ : $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function such that for almost all $z \in \Omega$, $f(z, 0) = 0$, $f(z,x) \geqslant 0$ for all $x > 0$, and

- (i) $f(z,x) \leqslant a(z)(1 + x^{r-1})$ for almost all $z \in \Omega$ and all $x \geqslant 0$, with $a \in L^\infty(\Omega)$, $2 < q < r < 2^*$;
- (ii) $\lim\limits_{x\to+\infty} \frac{f(z,x)}{x^{q-1}} = +\infty$ uniformly for almost all $z \in \Omega$, and the mapping $x \mapsto \frac{f(z,x)}{x}$ is nondecreasing on $(0, +\infty)$ for almost all $z \in \Omega$;
- (iii) $0 \leqslant \liminf\limits_{x\to 0^+} \frac{f(z,x)}{x} \leqslant \limsup\limits_{x\to 0^+} \frac{f(z,x)}{x} \leqslant \hat{\eta}$ uniformly for almost all $z \in \Omega$;
- (iv) for every $\rho > 0$, there exists $\hat{\xi}_\rho > 0$ such that for almost all $z \in \Omega$ the function $x \mapsto \hat{\xi}_\rho x - f(z,x)$ is nondecreasing on $[0, \rho]$.

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