Accepted Manuscript

Solving a singular beam equation by using a weak-form integral equation method

Chein-Shan Liu, Dongjie Liu, Wun-Sin Jhao

PII: S0893-9659(16)30245-2
DOI: http://dx.doi.org/10.1016/j.aml.2016.08.010
Reference: AML 5074

To appear in: *Applied Mathematics Letters*

Received date : 21 June 2016
Revised date : 16 August 2016
Accepted date : 17 August 2016

Please cite this article as: C. Liu, et al., Solving a singular beam equation by using a weak-form integral equation method, *Applied Mathematics Letters* (2016), http://dx.doi.org/10.1016/j.aml.2016.08.010

Solving a singular beam equation by using a weak-form integral equation method

Chein-Shan Liu[a], Dongjie Liu[b,*], Wun-Sin Jhao[c]

[a]Center for Numerical Simulation Software in Engineering and Sciences,
College of Mechanics and Materials, Hohai University, Nanjing, Jiangsu 210098, P.R. China

[b]Department of Mathematics, College of Sciences, Shanghai University, Shanghai, P.R. China

[c]Department of Civil Engineering, National Taiwan University, Taipei, Taiwan

[*]Corresponding author, E-mail: Liudj@shu.edu.cn

Abstract

For the fourth-order singular beam equation with tension dominated than the rigidity, we develop the most weak-form integral equation method (WFIEM) to find the singular solution. The WFIEM together with the exponentially and polynomially fitted trial solutions, which are designed to satisfy the boundary conditions automatically, can find very accurate numerical solution of the singular beam equation.

Keywords: Singular beam equation, Weak-form integral equation method, Exponentially/ polynomially fitted trial functions

1. Introduction

In this paper we propose a new numerical method for solving the fourth-order beam equation whose highest order derivative term is multiplied by a small perturbing parameter. When the boundary conditions are imposed to such type ordinary differential equations (ODEs), the resulting problems are usually called the singularly perturbed boundary value problems (SPBVPs). The most SPBVPs exhibit boundary layer behavior, which is a narrow region where the solution varies rapidly. The reader can refer [1, 2, 3, 4] for recently developed numerical methods to solve the fourth-order SPBVPs. Dong et al. [5] have discussed different weak-form formulations of the fourth-order ODEs under different boundary conditions and investigated the ill-posed properties of these problems.

The arrangement of this paper is given as follows. In Section 2, we propose a linear singularly perturbed problem of a fourth-order beam equation of which the tension force is dominated than the rigidity of the beam. Then in Section 3 we derive the most weak-form integral equation for the singular beam equation. In Section 4 we derive suitable test functions and trial functions by sine/polynomial fitted and exponential/polynomial fitted techniques. The linear system and scales are derived in Section 5, while the numerical examples are given in Section 6 to validate the effectiveness of the present method. Some conclusions are drawn in Section 7.