Applied Mathematics Letters 64 (2017) 87–93

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Consistent Riccati expansion solvability and soliton–cnoidal wave interaction solution of a $(2+1)$-dimensional Korteweg–de Vries equation

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A R T I C L E   I N F O

Article history:
Received 21 July 2016
Received in revised form 24 August 2016
Accepted 24 August 2016
Available online 1 September 2016

Keywords:
Consistent Riccati expansion method
$(2+1)$-dimensional KdV equation
Interaction solution

A B S T R A C T

In this paper, a $(2+1)$-dimensional KdV equation is investigated by using the consistent Riccati expansion (CRE) method proposed by Lou (2015). It is proved that the $(2+1)$-dimensional KdV equation is CRE solvable. Furthermore, soliton, cnoidal wave and soliton–cnoidal wave interaction solutions are obtained explicitly from different special solutions of the modified Schwarzian equation.

1. Introduction

It is of fundamental importance to study integrable property and exact solution of nonlinear evolution equations (NLEEs) in mathematical physics. Many effective methods have been constructed to find the exact solutions of the nonlinear systems, such as inverse scattering transformation, Darboux transformation, Hirota bilinear methods, Painlevé analysis, symmetry method, the variable separation approach and function expansion method. Recently, starting from nonlocal symmetries related to Darboux transformation, Bäcklund transformation and residual symmetries, various exact interaction solutions have been found through symmetry reduction for the different nonlinear integrable models [1–7]. Such interesting interaction solutions represent the complex excitation among different types of nonlinear excitations, which include solitons, cnoidal waves, Painlevé waves, Airy waves, Bessel waves, etc. Very recently, inspired from the novel results obtained via nonlocal symmetry method, Lou [8] proposed the consistent Riccati expansion (CRE)/consistent tanh expansion (CTE) method, which is a more direct but much simpler method. Based on this expansion method, a new solvability involving whether or not a original nonlinear system has a

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http://dx.doi.org/10.1016/j.aml.2016.08.016
0893-9659/

CRE is defined. Furthermore, it is shown that a lot of CRE solvable models possess quite similar interaction solutions between a soliton and other nonlinear waves [9–11].

In this paper, we consider the following (2 + 1)-dimensional KdV equation [12]

$$u_t + \alpha(u_{xxx} + 6uu_x) + \beta(4uu_y + 2u_x\partial_x^{-1}u_y + u_{xxy}) = 0, \tag{1}$$

where α and β are arbitrary constants. Peng [12] first constructed the (2 + 1)-dimensional KdV equation (1) by using Lax pair generating technique. Meanwhile, exact solutions and Bäcklund transformation were investigated by means of the singular manifold method in Ref. [12]. Eq. (1) includes several celebrated physical equations as special reductions. For $\alpha \neq 0$ and $\beta = 0$, Eq. (1) can be reduced to the (1+1)-dimensional KdV equation. For $\alpha = 0$ and $\beta \neq 0$, Eq. (1) becomes the Boiti–Leon–Pempinelli equation [13] or the Calogero–Bogoyavlenskii–Schiff equation [14]. With the aid of binary Bell polynomials, Wang et al. [15–17] investigated bilinear formalism, bilinear Bäcklund transformation, Lax pair, Darboux covariant Lax pair and the infinite conservation laws of Eq. (1). Wazwaz [18] established multiple soliton solutions and showed that the resonance phenomenon does not exist for the (2+1)-dimensional KdV equation (1). By using the WTC method, the Painlevé integrability of Eq. (1) was examined and the auto-Bäcklund transformation and several types of exact solutions were obtained by using the Painlevé truncated expansion method [19].

The outline of this paper is as follows. In Section 2, the consistent Riccati expansion method is applied to the (2 + 1)-dimensional KdV equation (1). It is shown that the (2 + 1)-dimensional KdV equation is CRE solvable. In Section 3, starting from the last consistent differential equation, one special form of solution is presented to construct exact solution for the original (2 + 1)-dimensional KdV equation. As a result, soliton, cnoidal wave and soliton–cnoidal wave interaction solutions are provided explicitly. Some conclusions and discussions are given in the last section.

2. Consistent Riccati expansion solvability for Eq. (1)

Let $v_x = u_y$, Eq. (1) is transformed into the following system

$$u_t + \alpha(u_{xxx} + 6uu_x) + \beta(4uu_y + 2u_xv + u_{xxy}) = 0, \tag{2}$$

$$v_x - u_y = 0. \tag{3}$$

According to the CRE method in Ref. [8], we get the form of solution

$$u = u_0 + u_1R(\phi) + u_2R(\phi)^2, \qquad v = v_0 + v_1R(\phi) + v_2R(\phi)^2, \tag{4}$$

where the function $R(\phi)$ need to satisfy the Riccati equation

$$R_\phi = a_0 + a_1R + a_2R^2, \quad R \equiv R(\phi). \tag{5}$$

Substituting (4) into Eqs. (2)–(3), and setting the coefficients of all the same powers of $R(\phi)$ to be zero, we obtain ten overdetermined differential equations with respect to only seven undetermined functions : $\{u_0, v_0, u_1, v_1, u_2, v_2, \phi\}$. After thorough analysis and quite tedious calculations, the general result reads:

$$u_2 = -2a_2^2\phi_x^2, \qquad v_2 = -2a_2^2\phi_x\phi_y, \qquad u_1 = -2a_2(\phi_{xx} + a_1\phi_x^2), \qquad v_1 = -2a_2(\phi_{xy} + a_1\phi_x\phi_y), \tag{6}$$

$$u_0 = -\frac{1}{4}(a_1^2 + 4a_0a_2)\phi_x^2 - a_1\phi_{xx} - \frac{1}{2}\frac{\phi_{xxx}}{\phi_x} + \frac{1}{4}\frac{\phi_{xx}^2}{\phi_x^2} + \lambda, \tag{7}$$

$$v_0 = \frac{1}{4}\frac{\alpha}{\beta}(a_1^2 - 4a_0a_2)\phi_x^2 - 2a_0a_2\phi_x\phi_y - a_1\phi_{xy} - 2\lambda\frac{\phi_y}{\phi_x} + \frac{3}{4}\frac{\alpha}{\beta}\frac{\phi_{xx}^2}{\phi_x^2} + \frac{\phi_{xx}\phi_{xy}}{\phi_x^2} - \frac{\phi_{xxy}}{\phi_x} - \frac{1}{2}\frac{\alpha}{\beta}\frac{\phi_{xxx}}{\phi_x} - \frac{1}{2\beta}\frac{\phi_t}{\phi_x} - 3\lambda\frac{\alpha}{\beta}, \tag{8}$$