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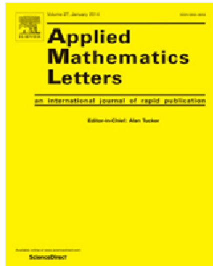

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Infinitely many periodic solutions for a class of new superquadratic second-order Hamiltonian systems*

Chun Li[1†] Ravi P. Agarwal[2] Daniel Paşca[3]

1. School of Mathematics and Statistics, Southwest University, Chongqing 400715, P. R. China
2. Department of Mathematics, Texas A&M University, Kingsville, TX 78363, USA
3. Department of Mathematics and Informatics, University of Oradea, Oradea 410087, Romania

Abstract

In this paper, we establish the existence of infinitely many periodic solutions for a class of new superquadratic second-order Hamiltonian systems. Our technique is based on the Fountain Theorem due to Bartsch.

Key words: Periodic solutions; Second-order Hamiltonian systems; Fountain Theorem

1 Introduction and main results

We consider the existence of infinitely many periodic solutions for the following second-order Hamiltonian systems

$$\begin{cases} \ddot{u}(t) + B(t)u(t) + \nabla F(t, u(t)) = 0, & a.e.\ t \in [0, T], \\ u(0) - u(T) = \dot{u}(0) - \dot{u}(T) = 0, \end{cases} \tag{1}$$

where $T > 0$, $B(t)$ is an $N \times N$ symmetric matrix, continuous and T-periodic in t. $F : [0, T] \times \mathbb{R}^N \to \mathbb{R}$ is T-periodic in t for all $x \in \mathbb{R}^N$ and satisfies the following assumption:

(**A**) $F(t, x)$ is measurable in t for each $x \in \mathbb{R}^N$ and continuously differential in x for $a.e.\,t \in [0, T]$ and there exist $a \in C(\mathbb{R}^+, \mathbb{R}^+)$, $b \in L^1(0, T; \mathbb{R}^+)$ such that

$$|F(t, x)| \le a(|x|)b(t), \quad |\nabla F(t, x)| \le a(|x|)b(t)$$

for all $x \in \mathbb{R}^N$ and $a.e.\,t \in [0, T]$, where $\nabla F(t, x)$ denotes the gradient of $F(t, x)$ in x.

In 1978, Rabinowitz [11] published his pioneer paper for the existence of periodic solutions for problem (1) under the following Ambrosetti-Rabinowitz superquadratic condition: there exist $\mu > 2$ and $L^* > 0$ such that

$$0 < \mu F(t, x) \le (\nabla F(t, x), x), \quad \forall\ |x| \ge L^*,\ a.e.\ t \in [0, T]. \tag{2}$$

From then on, various conditions have been applied to study the the existence and multiplicity of periodic solutions for Hamiltonian systems by using the critical point theory, see [2, 5–17, 19] and references therein.

Over the last few decades, many researchers have successfully replaced the Ambrosetti-Rabinowitz superquadratic condition (2) by other superquadratic conditions. Especially, when $B(t) \equiv 0$, Fei [6] studied the existence of periodic solutions for problem (1) under a class of new superquadratic condition: there is $\tau > 1$ such that

$$\liminf_{|x| \to \infty} \frac{(\nabla F(t, x), x) - 2F(t, x)}{|x|^\tau} > 0 \quad \text{uniformly for } a.e.\ t \in [0, T]. \tag{3}$$

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†Corresponding author. Tel.: +86 23 68253135; fax: +86 23 68253135. E-mail address: Lch1999@swu.edu.cn (C. Li).